Section 25.4

(5) $\int_1^4 x^{5/2}\,dx$

$= \frac{2}{7} x^{7/2} \Big|_1^4$

$= \frac{2}{7}\left(4^{7/2} - 1\right)$

$= \frac{2}{7}(127)$

$= \frac{254}{7}$

(7) $\int_3^6 \left(\frac{1}{\sqrt{x}} - 7\right) dx$

$= \int_3^6 \left(x^{-1/2} - 7\right) dx$

$= \left[2x^{1/2} - 7x\right]_3^6$

$= \left(2\sqrt{6} - 42\right) - \left(2\sqrt{3} - 21\right)$

$= 2\sqrt{6} - 2\sqrt{3} - 21$

(11) $\displaystyle\int_{-2}^{2} (T-2)(T+2)\,dT$

$$= \int_{-2}^{2} (T^2-4)\,dT$$

$$= \left[\frac{T^3}{3} - 4T\right]_{-2}^{2}$$

$$= \left(\frac{8}{3} - 8\right) - \left(\frac{-8}{3} + 8\right)$$

$$= \frac{16}{3} - 16$$

$$= \frac{-32}{3}$$

(15) $\displaystyle\int_{0}^{4} (1-\sqrt{x})^2\,dx$

$$= \int_{0}^{4} (1 - 2\sqrt{x} + x)\,dx$$

$$= \int_{0}^{4} (1 - 2x^{1/2} + x)\,dx$$

$$= \left[x - \frac{4}{3}x^{3/2} + \frac{x^2}{2}\right]_{0}^{4}$$

$$= \left(4 - \frac{4}{3}(8) + \frac{16}{2}\right) - 0$$

$$= \frac{4}{3}$$

(17) $\displaystyle\int_{-2}^{-1} 2x(4-x^2)^3\,dx$

$u = 4 - x^2$
$du = -2x\,dx$
$-du = 2x\,dx$
$x = -2 \Rightarrow u = 0$
$x = -1 \Rightarrow u = 3$

$$= -\int_0^3 u^3\,du$$

$$= -\left[\frac{u^4}{4}\right]_0^3$$

$$= -\left[\frac{81}{4} - 0\right]$$

$$= -\frac{81}{4}$$

(19) $\displaystyle\int_0^4 \frac{x\,dx}{\sqrt{x^2+9}}$

$$\boxed{\begin{aligned} u &= x^2+9 \\ du &= 2x\,dx \\ \frac{du}{2} &= x\,dx \\ x=0 &\Rightarrow u=9 \\ x=4 &\Rightarrow u=25 \end{aligned}}$$

$$= \frac{1}{2}\int_9^{25} \frac{du}{\sqrt{u}}$$

$$= \frac{1}{2}\int_9^{25} u^{-1/2}\,du$$

$$= \frac{1}{2}\left[2u^{1/2}\right]_9^{25}$$

$$= \left[u^{1/2}\right]_9^{25}$$

$$= 5-3$$

$$= 2$$

(21) $\displaystyle\int_{2.75}^{3.25} \frac{dx}{\sqrt[3]{6x+1}}$

$$\boxed{\begin{aligned} u &= 6x+1 \\ du &= 6\,dx \\ \frac{du}{6} &= dx \\ x = 2.75 &\Rightarrow u = 17.5 \\ x = 3.25 &\Rightarrow u = 20.5 \end{aligned}}$$

$$= \frac{1}{6}\int_{17.5}^{20.5} \frac{du}{\sqrt[3]{u}}$$

$$= \frac{1}{6}\int_{17.5}^{20.5} u^{-1/3}\,du$$

$$= \frac{1}{6}\left[\frac{3}{2}u^{2/3}\right]_{17.5}^{20.5}$$

$$= \frac{1}{4}\left[u^{2/3}\right]_{17.5}^{20.5}$$

$$= \frac{1}{4}\left(20.5^{2/3} - 17.5^{2/3}\right)$$

$$\approx 0.19$$

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$$\int_1^3 \frac{12x\,dx}{(2x^2+1)^3}$$

$$\boxed{\begin{aligned} u &= 2x^2+1 \\ du &= 4x\,dx \\ \frac{du}{4} &= x\,dx \\ x=1 &\Rightarrow u=3 \\ x=3 &\Rightarrow u=19 \end{aligned}}$$

$$= \frac{12}{4}\int_3^{19} \frac{du}{u^3}$$

$$= 3\int_3^{19} u^{-3}\,du$$

$$= 3\left[-\frac{1}{2}u^{-2}\right]_3^{19}$$

$$= -\frac{3}{2}\left[u^{-2}\right]_3^{19}$$

$$= \frac{-3}{2}\left[\frac{1}{361} - \frac{1}{9}\right]$$

$$= \frac{-3}{2}\left[\frac{-352}{3249}\right]$$

$$= \frac{176}{1083}$$

(25) $$\int_3^7 \sqrt{16t^2+8t+1}\, dt$$

$$= \int_3^7 \sqrt{(4t+1)^2}\, dt$$

$$= \int_3^7 (4t+1)\, dt$$

$$= \left[2t^2+t\right]_3^7$$

$$= 105-21$$

$$= 84$$

(27) $\int_0^2 2x(9-2x^2)^2\,dx$

$$u = 9-2x^2$$
$$du = -4x\,dx$$
$$-\frac{1}{4}du = x\,dx$$
$$x=0 \Rightarrow u=9$$
$$x=2 \Rightarrow u=1$$

$$= \frac{-2}{4}\int_9^1 u^2\,du$$

$$= -\frac{1}{2}\left[\frac{u^3}{3}\right]_9^1$$

$$= -\frac{1}{2}\left[\frac{1}{3} - \frac{729}{3}\right]$$

$$= -\frac{1}{2}\left[-\frac{728}{3}\right]$$

$$= \frac{364}{3}$$

(29)

$$\int_{-1}^{2} \frac{8x-2}{(2x^2-x+1)^3}\,dx$$

$$= 2\int_{-1}^{2} \frac{4x-1}{(2x^2-x+1)^3}\,dx$$

$$= 2\int_{4}^{7} \frac{du}{u^3}$$

$$= 2\int_{4}^{7} u^{-3}\,du$$

$$= 2\left[\frac{-1}{2}u^{-2}\right]_4^7$$

$$= -\left[u^{-2}\right]_4^7$$

$$= -\left[\frac{1}{49}-\frac{1}{16}\right]$$

$$= -\left[\frac{-33}{784}\right]$$

$$= \frac{33}{784}$$

$$\boxed{\begin{aligned} u &= 2x^2-x+1 \\ du &= (4x-1)\,dx \\ x=-1 &\Rightarrow u=4 \\ x=2 &\Rightarrow u=7 \end{aligned}}$$

(33)

$$\int_{\sqrt{5}}^{3} 2z \cdot \sqrt[4]{z^4+8z^2+16}\, dz$$

$$= \int_{\sqrt{5}}^{3} 2z \cdot \sqrt[4]{(z^2+4)^2}\, dz$$

$$= \int_{\sqrt{5}}^{3} 2z \cdot (z^2+4)^{1/2}\, dz$$

$$\boxed{\begin{array}{c} u = z^2+4 \\ du = 2z\,dz \\ z=\sqrt{5} \Rightarrow u = 9 \\ z = 3 \Rightarrow u = 13 \end{array}}$$

$$= \int_{9}^{13} u^{1/2}\, du$$

$$= \left[\frac{2}{3} u^{3/2}\right]_9^{13}$$

$$= \frac{2}{3}\left(13^{3/2} - 9^{3/2}\right)$$

$$= \frac{2}{3}\left(13\sqrt{13} - 27\right)$$

(41) $\int_{-1}^{1} t^{2k}\,dt$ k: a positive integer

$$= \left. \frac{t^{2k+1}}{2k+1} \right|_{-1}^{1}$$

$$\boxed{\text{Note: } (-1)^{\text{odd}} = -1}$$

$$= \frac{1}{2k+1} - \left(\frac{-1}{2k+1}\right)$$

$$= \frac{2}{2k+1}$$