(3) $x^2 - 4y^2 = 9$

Need implicit differentiation to find $\frac{dy}{dx}$.

$$2x - 8y\frac{dy}{dx} = 0$$

$$-8y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{x}{4y}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(5,2)} = \frac{5}{8}$$

$$m_{tan} = \frac{5}{8}$$

$$m_{normal} = \frac{-8}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{5}(x - 5)$$

$$5y - 10 = -8(x-5)$$

$$5y - 10 = -8x + 40$$

$$8x + 5y - 50 = 0$$

(5) $y = (x^2+3)^{1/2}$

$y' = \frac{1}{2}(x^2+3)^{-1/2}(2x)$

$= \dfrac{x}{\sqrt{x^2+3}}$

Set $\dfrac{x}{\sqrt{x^2+3}} = \dfrac{1}{2}$ (Call this equation $*$)

$x = \frac{1}{2}\sqrt{x^2+3}$

$2x = \sqrt{x^2+3}$

Square Both Sides : $4x^2 = x^2+3$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

Check these in $*$

$x = 1$ ✓

$x = -1$ ✗

$\boxed{\text{So } x = 1}$

$x = 1 \rightarrow y = \sqrt{x^2+3}$

$y = 2$

$\boxed{\text{Sub } x_1 = 1 \quad y_1 = 2 \quad m_{tan} = \frac{1}{2}}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x-1)$

$2y - 4 = x - 1$

$-x + 2y - 3 = 0$ or $x - 2y + 3 = 0$

⑦ $x = t^{1/2} + t \qquad y = \frac{1}{12} t^3$

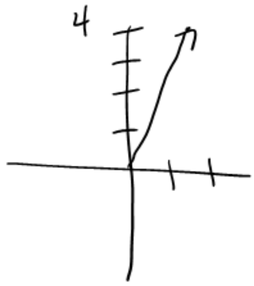
$v_x = \frac{1}{2} t^{-1/2} + 1 \qquad v_y = \frac{1}{4} t^2$

@ $t=4$: $v_x = 1.25 \qquad v_y = 4$

$$v = \sqrt{1.25^2 + 4^2}$$

$$\approx 4.19 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{4}{1.25}\right) \quad (+180°?)$$



$$\approx 72.6°$$

(13) $f(x) = x^3 - 3x^2 - x + 2$

x	$f(x)$
0	2
1	-1

Choose $x_0 = 1$

$f(x) = x^3 - 3x^2 - x + 2$

$f'(x) = 3x^2 - 6x - 1$

x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	
1	-1	-4	0.75	(0.75)
0.75	-0.0156	-3.8125	0.7459	(0.75)

Table to 4 decimal places.
Answer to 2 decimal places.

$x \approx 0.75$

(21) Sketch $y = x^4 - 32x$

a) y-intercept

$x = 0 \Rightarrow y = 0$

$(0,0)$

b) Relative Max/Min

$y' = 4x^3 - 32$

Set $y' = 0$: $4x^3 - 32 = 0$

$4x^3 = 32$

$x^3 = 8$

$x = 2$

		2	
y'	$\ominus$		$\oplus$
y	Dec		Inc

Relative Min at $(2, \underset{\uparrow \; y|_{x=2}}{-48})$

c) Points of Inflection

$y'' = 12x^2$

Set $y'' = 0$: $12x^2 = 0$

$x^2 = 0$

$x = 0$

		0	
y''	$\oplus$	0	$\oplus$
y	CU		CU

y is concave up for all x (no points of inflection)

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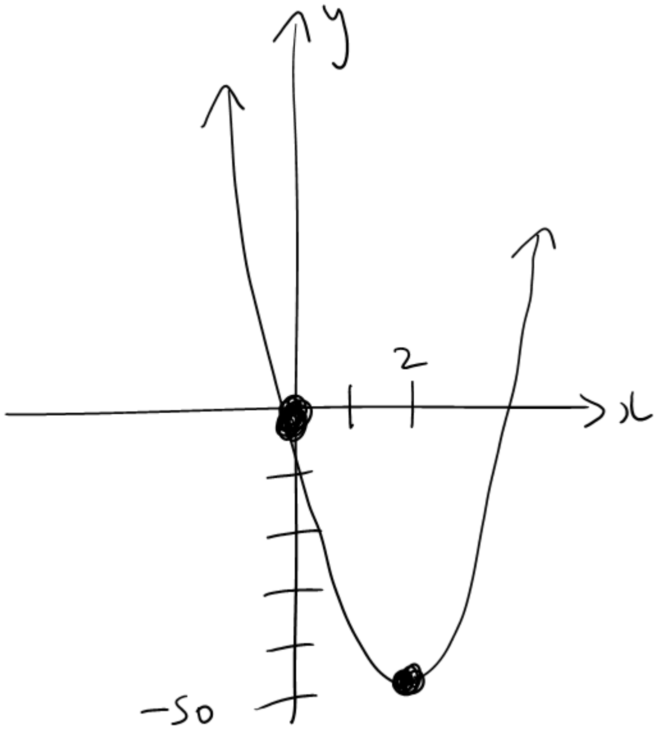
Cont'd



(25) $y = 4x^3 + x^{-1}$

$dy = (12x^2 - x^{-2})\,dx$

or $dy = \left(12x^2 - \frac{1}{x^2}\right)dx$

(31) $f(x) = (x^4 + 3x^2 + 8)^{1/2}$

$f'(x) = \frac{1}{2}(x^4 + 3x^2 + 8)^{-1/2}(4x^3 + 6x)$

$f(2) = 36^{1/2} = 6$

$f'(2) = \frac{1}{2}\left(\frac{1}{6}\right)(44) = \frac{44}{12} = \frac{11}{3}$

$f(x) \approx f(a) + f'(a)(x-a)$

$\approx f(2) + f'(2)(x-2)$

$\approx 6 + \frac{11}{3}(x-2)$ ✓

OR $\approx \frac{18}{3} + \frac{1}{3}(11x - 22)$

$\approx \frac{1}{3}[18 + 11x - 22]$

$\approx \frac{1}{3}(11x - 4)$ ✓

(39) Evaluate 3.02^5

First find the linearization of x^5 at $a=3$.

$$f(x) = x^5$$
$$f'(x) = 5x^4$$
$$f(3) = 243$$
$$f'(3) = 5(81) = 405$$

$$\begin{aligned} f(x) &\approx f(a) + f'(a)(x-a) \\ &\approx f(3) + f'(3)(x-3) \\ &\approx 243 + 405(x-3) \end{aligned}$$

So $x^5 \approx 243 + 405(x-3)$

Now sub $x = 3.02$:

$$\begin{aligned} 3.02^5 &\approx 243 + 405(0.02) \\ &\approx 251.1 \end{aligned}$$

(45) $x = 8.0t \qquad y = -0.15t^2$

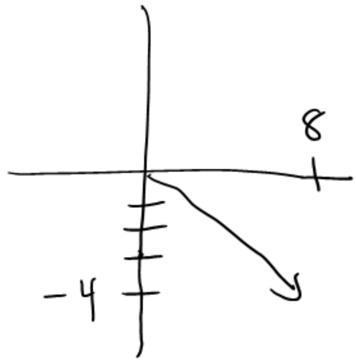
$v_x = 8.0 \qquad v_y = -0.30t$

@ $t=12$: $\quad v_x = 8.0 \qquad v_y = -3.6$

$$v = \sqrt{8.0^2 + (-3.6)^2}$$

$$\approx 8.8 \text{ m/s}$$

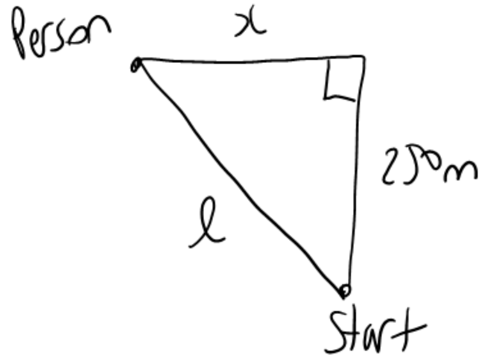
$$\theta = \tan^{-1}\left(\frac{-3.6}{8.0}\right) \quad (+180°?)$$



$\approx -24°$

or $336°$

(47)



Find $\frac{dl}{dt}$ if $\frac{dx}{dt} = 1 \frac{m}{s}$ and $t = 1$ min.

$$l^2 = x^2 + 250^2$$

Take $\frac{d}{dt}$:

$$2l \frac{dl}{dt} = 2x \frac{dx}{dt}$$

Missing Values:
Need x and l at $t = 1$ min

x: distance = speed · time

$$x = 1 \frac{m}{s} \cdot 60 \text{ s}$$

$$= 60\text{m}$$

l: $$l = \sqrt{60^2 + 250^2}$$

$$\approx 257.1 \text{ m}$$

$$2(257.1) \frac{dl}{dt} = 2(60)(1)$$

$$\frac{dl}{dt} \approx 0.23 \frac{m}{s}$$

(53) $P = 0.030r^3 - 2.6r^2 + 71r - 200$

where $6.0 \le r \le 30 \ \frac{m^3}{s}$

Find r that maximizes P.

$$\frac{dP}{dr} = 0.090r^2 - 5.2r + 71$$

Set $\frac{dP}{dr} = 0$: $\quad 0.090r^2 - 5.2r + 71 = 0$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5.2 \pm \sqrt{1.48}}{0.180}$$

$$\approx 36, 22$$

$\boxed{\text{Recall } 6.0 \le r \le 30 \ \frac{m^3}{s}}$

The value of r that maximizes P is $r \approx 22 \ \frac{m^3}{s}$.

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$A = \pi r^2$

$\frac{dr}{dt} = 15 \ \frac{m}{min}$

Find $\frac{dA}{dt}$ when $r = 400m$.

$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$$

$$= 2\pi r \ \frac{dr}{dt}$$

$$= 2\pi\,(400)\,(15)$$

$$\approx 38{,}000 \ \frac{m^2}{min}$$

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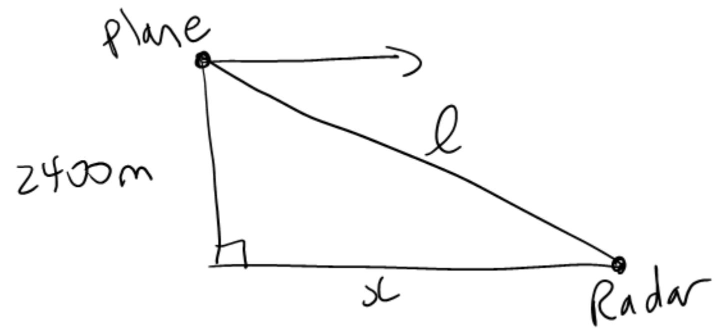


$$\frac{dl}{dt} = -1110 \ \frac{km}{h}$$

Find $\frac{dx}{dt}$ when $x = 8000$ m.

$$l^2 = x^2 + 2400^2$$

Take $\frac{d}{dt}$: $\quad 2l\frac{dl}{dt} = 2x\frac{dx}{dt}$

Missing Value:

Find l when $x = 8000$m

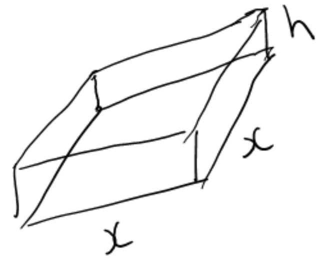
$$l = \sqrt{8000^2 + 2400^2}$$
$$\approx 8352 \text{ m}$$

$$2(8352)(-1110) = 2(8000)\frac{dx}{dt}$$

$$\frac{dx}{dt} = -1160 \ \frac{km}{h}$$

OR Plane's speed is $1160 \ \frac{km}{h}$.

(69)



$1\ dm = 0.1\ m$

$1\ dm^2 = (0.1\ m)(0.1\ m)$

$= 0.01\ m^2$

$27\ dm^2 = 0.27\ m^2$

1) Maximize Volume $f = x^2 h$

Restriction: $x^2 + 4xh = 0.27\ m^2$

2) Single-Variable

$$4xh = 0.27 - x^2$$

$$h = \frac{0.27 - x^2}{4x}$$

$$h = \frac{0.27 - x^2}{4x} \rightarrow f = x^2 h$$

$$= x^2\left(\frac{0.27 - x^2}{4x}\right)$$

$$= \frac{x(0.27 - x^2)}{4}$$

$$= 0.0675x - 0.25x^3$$

3) Critical Points

$$f' = 0.0675 - 0.75x^2$$

Set $f' = 0$:

$$0.0675 - 0.75x^2 = 0$$

$$0.0675 = 0.75x^2$$

$$0.09 = x^2$$

$$x = \pm 0.3$$

$$x > 0 \Rightarrow x = 0.3$$

4) Answer

The maximim volume is $f = 0.0675x - 0.25x^3$

$= 0.0135\ m^3\ (13.5\ dm^3)$

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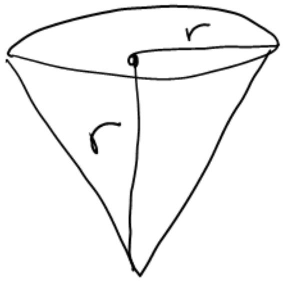

$V = \frac{1}{3}\pi r^2 h$.

But $h = r$ always.

So $V = \frac{1}{3}\pi r^3$.

$$\frac{dV}{dt} = 3.00 \ \frac{m^3}{min}$$

Find $\frac{dr}{dt}$ when $r = 2.50 m$.

$$V = \frac{1}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$$

$$\frac{dV}{dt} = \pi r^2 \frac{dr}{dt}$$

$$3.00 = \pi (2.50)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} \approx 0.153 \quad \frac{m}{min}$$

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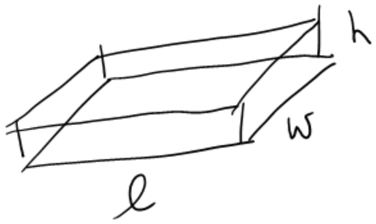

$V = 38.3 \text{ m}^3$

$\boxed{w = 0.75l}$

$lwh = 38.3$

$0.75l^2h = 38.3$

$\boxed{h = \dfrac{38.3}{0.75l^2}}$

Minimize cost $f = 6wl + 9(2wh + 2lh) + 4.5wl$

$= 10.5wl + 18wh + 18lh$

$= 10.5(0.75l)l + 18(0.75l)\left(\dfrac{38.3}{0.75l^2}\right) + 18l\left(\dfrac{38.3}{0.75l^2}\right)$

$= 7.875l^2 + \dfrac{1608.6}{l}$

$f' = 15.75l - \dfrac{1608.6}{l^2}$

Set $f' = 0$: $15.75l - \dfrac{1608.6}{l^2} = 0$

$15.75l = \dfrac{1608.6}{l^2}$

$l^3 \approx 102.13$

$l \approx 4.67$

The dimensions that minimize cost are:

$l \approx 4.67\text{m}$ $w \approx 3.50\text{m}$ $h \approx 2.34\text{m}$