③ Maximize $s = 34.3t - 4.9t^2$

$$s' = 34.3 - 9.8t$$

Set $s' = 0$: $$34.3 - 9.8t = 0$$
$$-9.8t = -34.3$$
$$t = 3.5$$

The maximum height is
$$s(3.5) = 34.3(3.5) - 4.9(3.5)^2$$
$$\approx 60\,m$$

(9) Maximize $S = 360A - 0.10A^3$

$S' = 360 - 0.30A^2$

Set $S' = 0$: $360 - 0.30A^2 = 0$

$-0.30A^2 = -360$

$A^2 = 1200$

$A \approx \pm 35$

But area > 0 $A \approx 35$

The area that maximizes savings is $A \approx 35\ m^2$.

The maximum savings is

$S(35) = 360(35) - 0.10(35)^3$

$\approx \$8300$

(15) Minimize $v = k\sqrt{\frac{l}{a} + \frac{a}{l}}$

Note: a and k are positive constants.

$$v = k\left(\frac{l}{a} + a l^{-1}\right)^{1/2}$$

$$v' = \frac{1}{2}k\left(\frac{l}{a} + a l^{-1}\right)^{-1/2}\left(\frac{1}{a} - a l^{-2}\right)$$

Set $v' = 0$:

$$\frac{1}{2}k\left(\frac{l}{a} + a l^{-1}\right)^{-1/2}\left(\frac{1}{a} - a l^{-2}\right) = 0$$

$$\frac{k\left(\frac{1}{a} - a l^{-2}\right)}{2\sqrt{\frac{l}{a} + a l^{-1}}} = 0$$

$$k\left(\frac{1}{a} - a l^{-2}\right) = 0$$

$$\frac{1}{a} - a l^{-2} = 0$$

$$\frac{1}{a} - \frac{a}{l^2} = 0$$

$$\frac{1}{a} = \frac{a}{l^2}$$

$$l^2 = a^2$$

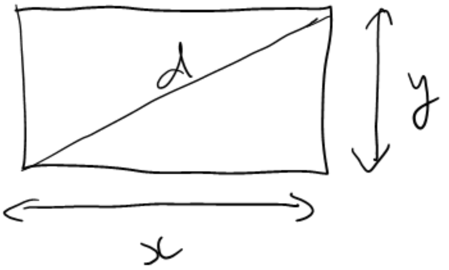
$$l = \pm a$$

But $l > 0$

$$l = a$$

The length of the wave that results in minimum velocity is $l = a$.

(17)



1) Minimize $d = \sqrt{x^2 + y^2}$

Restriction : $2x + 2y = 48$

It's equivalent to minimize $f = d^2$
$= x^2 + y^2$

(Minimizing d^2 and minimizing d give the same x-value and y-value).

$$\boxed{\begin{array}{l} \text{So minimize } f = x^2 + y^2 \\ \text{Restriction : } 2x + 2y = 48 \end{array}}$$

2) Single Variable

$$2x + 2y = 48$$
$$2y = 48 - 2x$$
$$y = 24 - x$$

$y = 24 - x \rightarrow$ $f = x^2 + y^2$

$$f = x^2 + (24 - x)^2$$

3) Critical Points

$$f' = 2x + 2(24 - x)(-1)$$

Set $f' = 0$: $\quad 2x \; - 2(24 - x) = 0$

$\rightarrow$

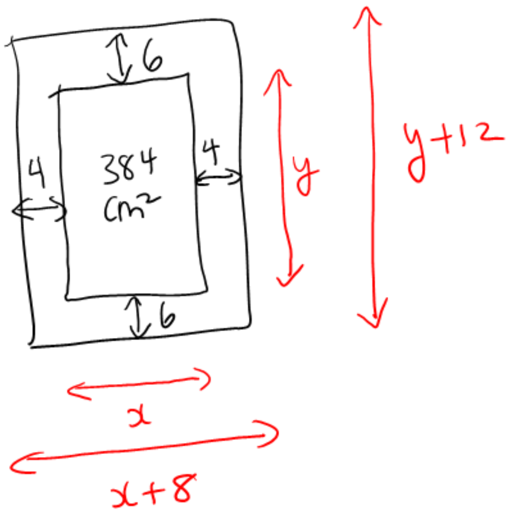
17
Cont'd

$$2x - 2(24-x) = 0$$
$$2x - 48 + 2x = 0$$
$$4x = 48$$
$$x = 12$$

$$x = 12 \rightarrow y = 24 - x$$
$$y = 12$$

4) Answer The dimensions that minimize the length of the diagonal are 12cm x 12cm.

(29)



1) Minimize area $f = (x+8)(y+12)$

Restriction : $xy = 384$

2) Single Variable

$$xy = 384$$
$$y = \frac{384}{x}$$

$y = \frac{384}{x} \rightarrow f = (x+8)(y+12)$

$$= (x+8)\left(\frac{384}{x} + 12\right)$$
$$= 384 + 12x + \frac{3072}{x} + 96$$

3) Critical Points

$$f' = 12 - \frac{3072}{x^2}$$

Set $f' = 0$: $\quad 12 - \frac{3072}{x^2} = 0$

$$12 = \frac{3072}{x^2}$$

$$12x^2 = 3072$$
$$x^2 = 256$$

$\rightarrow$

29
Cont'd

$$x^2 = 256$$
$$x = \pm 16$$
But $x > 0$ $\quad x = 16$

$$x = 16 \rightarrow y = \frac{384}{x} = \frac{384}{16} = 24$$

Recall the poster's dimensions are
$x + 8 = 24$ and $y + 12 = 36$

4) Answer
The dimensions of the poster with the smallest area are 24cm × 36cm.

(35) Curve $y = 6x^2 - x^3$

Slope $y' = 12x - 3x^2$

Maximize $f = 12x - 3x^2$

$f' = 12 - 6x$

Set $f' = 0$: $12 - 6x = 0$

$12 = 6x$

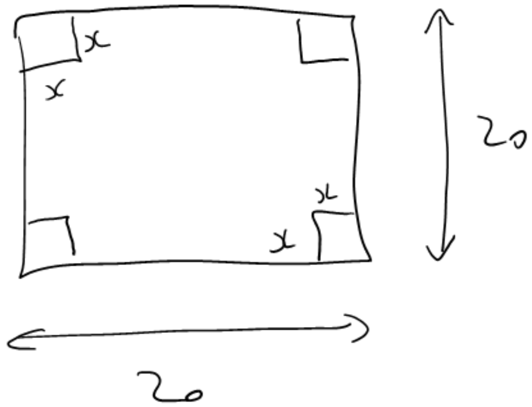
$x = 2$

$x = 2 \rightarrow f = 12x - 3x^2$

$= 12$

The maximum slope is 12.

(43)



Maximize volume $f = (20-2x)(20-2x)x$

$$f = x(20-2x)^2$$

$$f' = x\left[2(20-2x)(-2)\right] + (20-2x)^2(1)$$

$$= -4x(20-2x) + (20-2x)^2$$

FACTOR $= (20-2x)(-4x + 20 - 2x)$

$$= (20-2x)(20-6x)$$

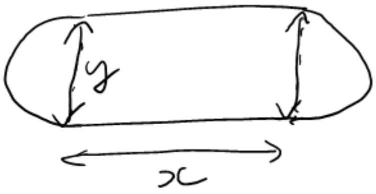
Set $f' = 0$: $(20-2x)(20-6x) = 0$

$x = 10\text{ cm}$ → MIN. VOLUME because $V = 0$

$x \approx 3.3\text{ cm}$ → MAX. VOLUME

The side of the square that is to be cut out must be 3.3 cm.

(45)



1) Maximize rectangular area $f = xy$

Restriction: perimeter $= 400$

$$2x + (\text{circumference of circle}) = 400$$
$$2x + (\pi \cdot \text{diameter}) = 400$$
$$2x + \pi y = 400$$

2) Single Variable

$$2x + \pi y = 400$$
$$\pi y = 400 - 2x$$
$$y = \frac{400 - 2x}{\pi}$$

$$y = \frac{400-2x}{\pi} \rightarrow f = xy$$
$$= \frac{x(400-2x)}{\pi}$$
$$= \frac{400x - 2x^2}{\pi}$$

3) Critical Points

$$f' = \frac{400 - 4x}{\pi}$$

Set $f' = 0$:

$$\frac{400 - 4x}{\pi} = 0$$
$$400 - 4x = 0$$
$$x = 100$$

4) Answer

To maximize the rectangular area, $x = 100$ m.