(3) $y = 5x^2 - 4x$

Take $\frac{d}{dt}$: $\frac{dy}{dt} = (10x - 4)\frac{dx}{dt}$

Sub $\frac{dx}{dt} = 0.5$ and $x = 5$

$\frac{dy}{dt} = 46(0.5)$

$= 23$

(7) $y = \frac{2}{x+1}$

$y = 2(x+1)^{-1}$

$y' = -2(x+1)^{-2}$

Take $\frac{d}{dt}$: $\frac{d}{dt}[y'] = [4(x+1)^{-3}(1)]\frac{dx}{dt}$

Sub $x = 3.0$ and $\frac{dx}{dt} = 0.5$ unit/s

$\frac{d}{dt}[y'] = 4(4)^{-3}(0.5)$

≈ 0.03 unit/s

(9) $v = 18\sqrt{T}$

$v = 18\,T^{1/2}$

Take $\frac{d}{dt}$: $\frac{dv}{dt} = \frac{dv}{dT}\frac{dT}{dt}$

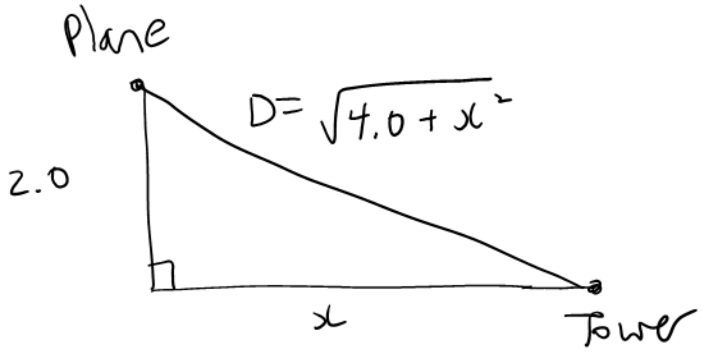
$= 9\,T^{-1/2}\frac{dT}{dt}$

Sub $T = 25\,N$ $\quad \frac{dT}{dt} = 0.20\ N/s$

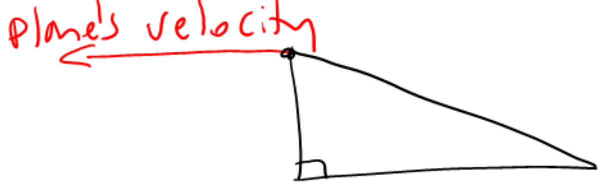
$\frac{dv}{dt} = 9(25)^{-1/2}(0.20)$

$= 0.36\ m/s$

(15)



Question assumes that plane is flying horizontally, away from the tower.



Find $\frac{dD}{dt}$ when $x = 6.2$ km and $\frac{dx}{dt} = 350$ km/h

$$D = \sqrt{4.0 + x^2}$$

Take $\frac{d}{dt}$:

$$\frac{dD}{dt} = \frac{dD}{dx}\frac{dx}{dt}$$

$$\frac{dD}{dt} = \frac{1}{2}(4.0 + x^2)^{-1/2}(2x)\frac{dx}{dt}$$

Sub $x = 6.2$, $\frac{dx}{dt} = 350$

$$\frac{dD}{dt} = \frac{1}{2}(4.0 + 6.2^2)^{-1/2}(12.4)(350)$$

$$\approx 330 \text{ km/h}$$

(17)

$$r = \sqrt{0.40\,\lambda}$$

$$r = (0.40\,\lambda)^{1/2}$$

$$\frac{dr}{dt} = \frac{dr}{d\lambda}\,\frac{d\lambda}{dt}$$

$$\frac{dr}{dt} = \frac{1}{2}(0.40\,\lambda)^{-1/2}(0.40)\,\frac{d\lambda}{dt}$$

Sub $\lambda = 6.0 \times 10^{-7}$

$$\frac{d\lambda}{dt} = 0.10 \times 10^{-7}$$

$$\frac{dr}{dt} = \frac{1}{2}(0.40 \times 6.0 \times 10^{-7})^{-1/2}(0.40)(0.10 \times 10^{-7})$$

$$\approx 4.1 \times 10^{-6}\ \text{m/s}$$

(19)

$$B = \frac{k}{\left[r^2 + \left(\frac{l}{2}\right)^2\right]^{3/2}}$$ Note: k, l are constants

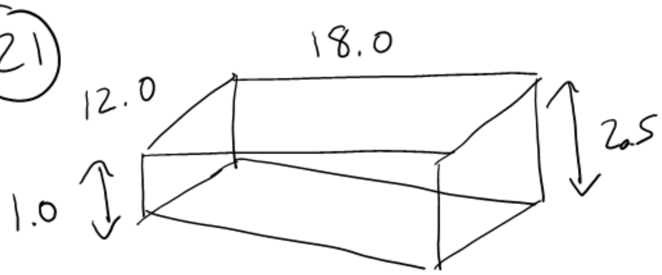
$$B = k\left[r^2 + \left(\frac{l}{2}\right)^2\right]^{-3/2}$$

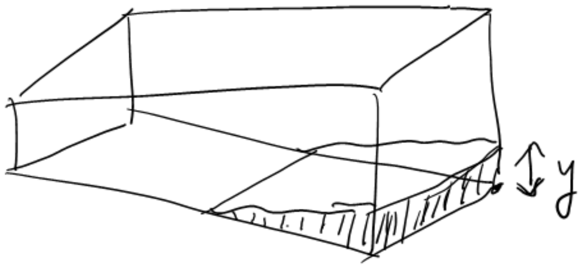
$$\frac{dB}{dt} = \frac{dB}{dr}\,\frac{dr}{dt}$$

$$= -\frac{3}{2}k\left[r^2 + \left(\frac{l}{2}\right)^2\right]^{-5/2}[2r]\,\frac{dr}{dt}$$

$$= \frac{-3kr}{\left[r^2 + \left(\frac{l}{2}\right)^2\right]^{5/2}}\,\frac{dr}{dt}$$

(21)

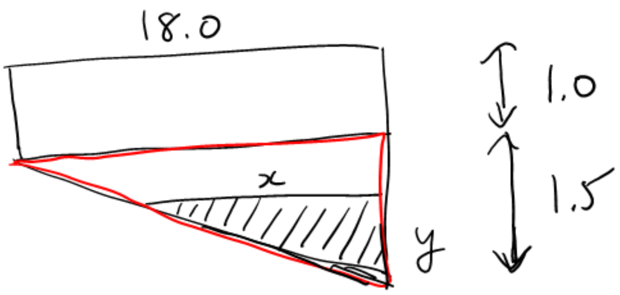




$$\frac{dV}{dt} = 0.80 \ \frac{m^3}{min}$$

Find $\frac{dy}{dt}$ when $y = 1.0$ m.

Find equation for V



Similar Triangles $\quad \frac{x}{y} = \frac{18.0}{1.5}$

$$x = 12y$$

$$V = (\text{triangular area})(\text{width of pool})$$

$$= \frac{1}{2} xy \ (12.0)$$

$$= \frac{1}{2} (12y) y \ (12.0)$$

$$= 72.0 \ y^2$$

$$\frac{dV}{dt} = \frac{dV}{dy} \frac{dy}{dt}$$

→ See next page

(21) Cont'd

$$\frac{dV}{dt} = \frac{dV}{dy}\frac{dy}{dt}$$

$V = 72.0y^2$

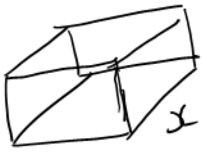
$$\frac{dV}{dt} = 144y\frac{dy}{dt}$$

Sub $\frac{dV}{dt} = 0.80$ $y = 1.0$

$$0.80 = 144\frac{dy}{dt}$$

$$\frac{dy}{dt} = 0.0056 \frac{m}{min}$$

(25)



$$\frac{dx}{dt} = -0.500 \ \frac{mm}{min}$$

Find $\frac{dV}{dt}$ when $x = 8.20$ mm.

$$V = x^3$$

$$\frac{dV}{dt} = \frac{dV}{dx} \frac{dx}{dt}$$

$$= 3x^2 \frac{dx}{dt}$$

Sub $x = 8.20$ $\quad \frac{dx}{dt} = -0.500$

$$\frac{dV}{dt} = -101 \ \frac{mm^3}{min}$$

(35) $w = 650\left(\frac{6400}{6400+h}\right)$

Find $\frac{dw}{dt}$ when $h = 1200$ km, $\frac{dh}{dt} = 6\ \frac{km}{s}$.

$$w = 650(6400)(6400+h)^{-1}$$

$$\frac{dw}{dt} = \frac{dw}{dh}\frac{dh}{dt}$$

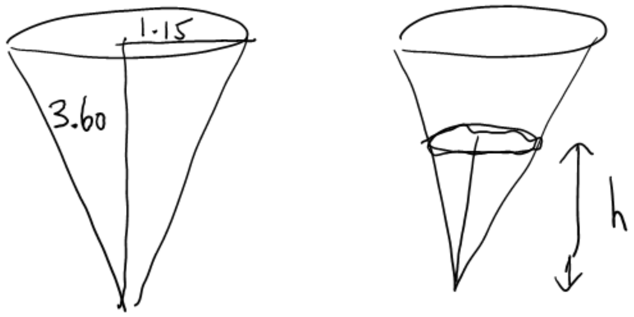
$$= -650(6400)(6400+h)^{-2}(1)\ \frac{dh}{dt}$$

Sub $h = 1200$, $\frac{dh}{dt} = 6$

$$\frac{dw}{dt} = -650(6400)(7600)^{-2}(6)$$

$$\approx -0.4\ \frac{N}{s}$$

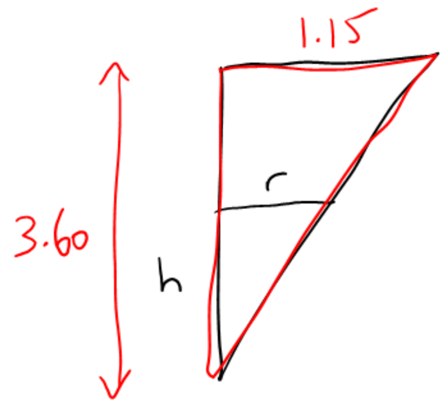
(37)



Find $\frac{dh}{dt}$ if $\frac{dV}{dt} = 0.500 \frac{m^3}{min}$, $h = 1.80 m$.

$V = \frac{1}{3}\pi r^2 h$

Similar Triangles



$$\frac{r}{h} = \frac{1.15}{3.60}$$

$$r = \frac{1.15}{3.60} h$$

$$V = \frac{1}{3}\pi\left(\frac{1.15}{3.60} h\right)^2 h$$

$$V = \frac{1}{3}\left(\frac{1.15}{3.60}\right)^2 \pi h^3$$

Take $\frac{d}{dt}$: $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$

$$\frac{dV}{dt} = \left(\frac{1.15}{3.60}\right)^2 \pi h^2 \frac{dh}{dt}$$ →

(37) Cont'd

$$\frac{dV}{dt} = \left(\frac{1.15}{3.60}\right)^2 \pi h^2 \frac{dh}{dt}$$

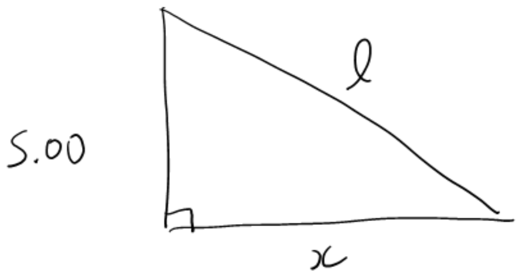
Sub $\frac{dV}{dt} = 0.500$, $h = 1.80$

$$0.500 = \left(\frac{1.15}{3.60}\right)^2 \pi (1.80)^2 \frac{dh}{dt}$$

$$\frac{0.500}{\pi (1.80)^2} \left(\frac{3.60}{1.15}\right)^2 = \frac{dh}{dt}$$

$$\frac{dh}{dt} \simeq 0.481 \ \frac{m}{min}$$

(41)



$$\frac{dl}{dt} = -2.50 \ \frac{m}{s}$$

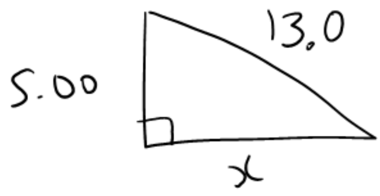
Find $\frac{dx}{dt}$ when $l = 13$.

$$l = \sqrt{5.00^2 + x^2}$$

$$\frac{dl}{dt} = \frac{dl}{dx} \frac{dx}{dt}$$

$$\frac{dl}{dt} = \frac{1}{2}\left(5.00^2 + x^2\right)^{-1/2}(2x)\ \frac{dx}{dt}$$

Missing value: Find x when $l = 13.0$



$$5.00^2 + x^2 = 13.0^2$$

$$x^2 = 144$$

$$x = \pm 12.0$$

$$x = 12.0$$

→

$$\frac{dl}{dt} = \frac{1}{2}\left(5.00^2 + x^2\right)^{-1/2}(2x)\,\frac{dx}{dt}$$

Sub $\frac{dl}{dt} = -2.50$, $x = 12.0$

$$-2.50 = \frac{1}{2}\left(5.00^2 + 12.0^2\right)^{-1/2}(24.0)\,\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-2.50\,(2)\left(5.00^2 + 12.0^2\right)^{1/2}}{24.0}$$

$$\approx -2.71\ \frac{m}{s}$$

The boat is approaching the wharf at $2.71\ \frac{m}{s}$.