(3)

$y = x^2 + 2$

$y' = 2x$

$y'\big|_{x=2} = 4$

$\boxed{m = 4 \qquad x_1 = 2 \qquad y_1 = 6}$

$y - y_1 = m(x - x_1)$

$y - 6 = 4(x - 2)$

$y - 6 = 4x - 8$

$y - 4x + 2 = 0$

or $-y + 4x - 2 = 0$

Use Wolfram Alpha to plot the curve and the tangent line.

Type: Plot y=x^2+2 and -y+4x-2=0

⑤ $y = \dfrac{1}{x^2+1}$

$y = (x^2+1)^{-1}$

$y' = -(x^2+1)^{-2}(2x)$

$y'\Big|_{x=1} = \dfrac{-1}{2^2}(2)$

$= -\dfrac{1}{2}$

$$\boxed{m = \frac{-1}{2} \quad x_1 = 1 \quad y_1 = \frac{1}{2}}$$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{-1}{2}(x-1)$

Multiply by -2 : $-2y + 1 = x - 1$

$-x - 2y + 2 = 0$

or $x + 2y - 2 = 0$

⑦ $y = 6x - 2x^2$

$y' = 6 - 4x$

$y'|_{x=2} = -2$

$m_{tan} = -2$

$m_{normal} = \frac{1}{2}$

$\boxed{m = \frac{1}{2} \quad x_1 = 2 \quad y_1 = 4}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{2}(x - 2)$

Multiply by 2: $2y - 8 = x - 2$

$-x + 2y - 6 = 0$

or $x - 2y + 6 = 0$

Use Wolfram Alpha.

Type: Plot y=6x-2x^2 and x-2y+6=0.

(9) $y = \frac{6}{(x^2+1)^2}$

$y = 6(x^2+1)^{-2}$

$y' = -12(x^2+1)^{-3}(2x)$

$y'|_{x=1} = \frac{-12}{2^3}(2)$

$= -3$

$m_{tan} = -3$

$m_{normal} = \frac{1}{3}$

$\boxed{m = \frac{1}{3} \quad x_1 = 1 \quad y_1 = \frac{3}{2}}$

$y - y_1 = m(x - x_1)$

$y - \frac{3}{2} = \frac{1}{3}(x-1)$

Multiply by 6: $6y - 9 = 2(x-1)$

$6y - 9 = 2x - 2$

$-2x + 6y - 7 = 0$

or $2x - 6y + 7 = 0$

(11) $y = x^2 - 2x$

$y' = 2x - 2$

$m_{tan} = 2x - 2$

Set $m_{tan} = 2$:

$2 = 2x - 2$

$4 = 2x$

$2 = x$

$x = 2 \rightarrow y = x^2 - 2x$

$y = 0$

$$\boxed{m = 2 \quad x_1 = 2 \quad y_1 = 0}$$

$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 2)$

$y = 2x - 4$

Use Wolfram Alpha to plot $y = x^2 - 2x$ and $y = 2x - 4$.

(13) $y = (2x-1)^3$

$y' = 3(2x-1)^2(2)$

$m_{tan} = 6(2x-1)^2$

Want $m_{normal} = -\frac{1}{24}$

$m_{tan} = 24$

Set $m_{tan} = 24$: $24 = 6(2x-1)^2$

$4 = (2x-1)^2$

$\pm 2 = 2x-1$

$2x-1 = 2$
$2x = 3$
$x = \frac{3}{2}$

$2x-1 = -2$
$2x = -1$
$x = -\frac{1}{2}$
Ignore because we were given $x > 0$

$x = \frac{3}{2} \rightarrow y = (2x-1)^3$

$y = 2^3$

$y = 8$

$$\boxed{m_{normal} = \frac{-1}{24} \quad x_1 = \frac{3}{2} \quad y_1 = 8}$$

$\rightarrow$

(13) Cont'd

$$y - y_1 = m(x - x_1)$$

$$y - 8 = \frac{-1}{24}\left(x - \frac{3}{2}\right)$$

Multiply by -24: $\quad -24y + 192 = x - \frac{3}{2}$

Multiply by 2: $\quad -48y + 384 = 2x - 3$

$$-2x - 48y + 387 = 0$$

or $\quad 2x + 48y - 387 = 0$

Use Wolfram Alpha.

Type: "Plot $y = (2x-1)\wedge 3$ and $2x + 48y - 387 = 0$ from $x=0$ to $x=3$"

(17) Rephrased: Find the tangent lines to $y = x + 2x^2 - x^4$ at $x = 1$ and $x = -1$. Show that they are the same line.

$$y = x + 2x^2 - x^4$$

$$y' = 1 + 4x - 4x^3$$

Tangent line at $x = 1$

$$y'\big|_{x=1} = 1$$

$m = 1 \quad x_1 = 1 \quad y_1 = 2$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x - 1)$$

$$y - 2 = x - 1$$

$$-x + y - 1 = 0$$

Tangent line at $x = -1$

$$y'\big|_{x=-1} = 1$$

$m = 1 \quad x_1 = -1 \quad y_1 = 0$

$$y - y_1 = m(x - x_1)$$

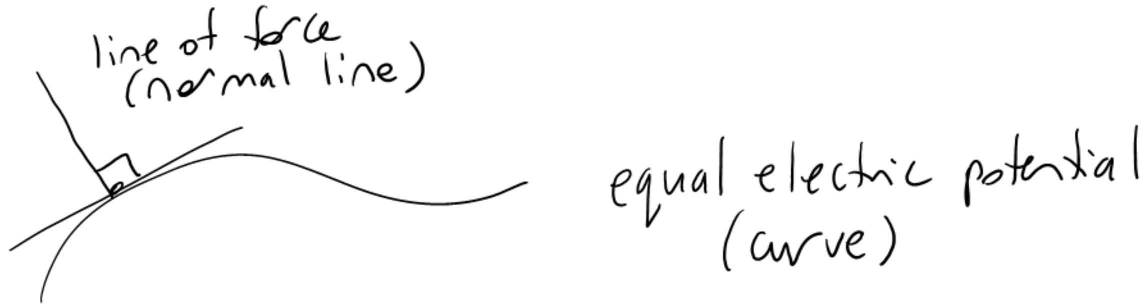
$$y - 0 = 1(x + 1)$$

$$y = x + 1$$

$$-x + y - 1 = 0$$

Same line ✓

(27)

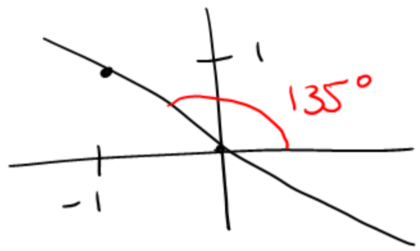


Given : $y = \sqrt{2x^2+8}$

Normal line has an inclination of 135°.

Find the normal line.

Notice that if a line is inclined at 135° then its slope is -1:



$$m_{normal} = -1$$

$$m_{tan} = 1$$

$$y = (2x^2+8)^{1/2}$$

$$y' = \frac{1}{2}(2x^2+8)^{-1/2}(4x)$$

$$m_{tan} = \frac{2x}{\sqrt{2x^2+8}}$$

Set $m_{tan} = 1$: $1 = \dfrac{2x}{\sqrt{2x^2+8}}$ →

(27) Cont'd

$$\sqrt{2x^2+8} = 2x$$

Square both sides : $2x^2+8 = 4x^2$

$$8 = 2x^2$$

$$4 = x^2$$

$$x = \pm 2$$

Because we squared both sides, some of these x-values may be extraneous. Check : $1 = \frac{2x}{\sqrt{2x^2+8}}$

$x=2$ | $x=-2$

$1 = \frac{4}{\sqrt{16}}$

$x=2$ is a solution ✓

$1 = \frac{-4}{\sqrt{16}}$

$x=-2$ is not a solution

$x=2 \rightarrow y = \sqrt{2x^2+8}$

$$y = 4$$

$$\boxed{m_{normal} = -1 \qquad x_1 = 2 \qquad y_1 = 4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -1(x-2)$$

$$-y+4 = 1(x-2)$$

$$-y+4 = x-2$$

$$-x-y+6 = 0$$

or $x+y-6=0$