(5)

$$\lim_{x \to 2} \frac{4x-8}{x^2-4}$$

$$= \lim_{x \to 2} \frac{4(x-2)}{(x-2)(x+2)}$$

$$= \lim_{x \to 2} \frac{4}{x+2}$$

$$= \frac{4}{4}$$

$$= 1$$

(7)

$$\lim_{x \to 3} \frac{x^2-5x+6}{x^2-2x-3}$$

$$= \lim_{x \to 3} \frac{(x-2)(x-3)}{(x+1)(x-3)}$$

$$= \lim_{x \to 3} \frac{x-2}{x+1}$$

$$= \frac{1}{4}$$

15

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{6-2(x+h)^2 - 6 + 2x^2}{h}$$

$$= \lim_{h\to 0} \frac{-2(x^2+2xh+h^2) + 2x^2}{h}$$

$$= \lim_{h\to 0} \frac{-2x^2-4xh-2h^2+2x^2}{h}$$

$$= \lim_{h\to 0} \frac{-4xh-2h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(-4x-2h)}{h}$$

$$= \lim_{h\to 0} -4x-2h$$

$$= -4x$$

(17) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2}{(x+h)^2} - \frac{2}{x^2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2x^2 - 2(x+h)^2}{(x+h)^2 x^2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2x^2 - 2(x^2+2xh+h^2)}{(x+h)^2 x^2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2x^2 - 2x^2 - 4xh - 2h^2}{(x+h)^2 x^2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{-4xh - 2h^2}{(x+h)^2 x^2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h} \cdot \frac{h(-4x-2h)}{(x+h)^2 x^2}$$

$$= \lim_{h \to 0} \frac{-4x-2h}{(x+h)^2 x^2}$$

$$= \frac{-4x}{x^4}$$

$$= \frac{-4}{x^3}$$

(19) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h+5} - \sqrt{x+5}\right)}{h} \frac{\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}{\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}$$

$$= \lim_{h \to 0} \frac{x+h+5 - (x+5)}{h\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h+5} + \sqrt{x+5}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+5} + \sqrt{x+5}}$$

$$= \frac{1}{2\sqrt{x+5}}$$

(23) $y = 4\sqrt{x} - \frac{3}{x} + \sqrt{3}$

$y = 4x^{1/2} - 3x^{-1} + \sqrt{3}$

$y' = 2x^{-1/2} + 3x^{-2}$

or $y' = \frac{2}{\sqrt{x}} + \frac{3}{x^2}$

Could also be written as $y' = \frac{1}{x^2}\left(2x^{3/2} + 3\right)$

(29)

$$y = \frac{3\pi}{(5-2x^2)^{3/4}}$$

$$y = 3\pi(5-2x^2)^{-3/4}$$

$$y' = 3\pi \cdot \left(\frac{-3}{4}\right)(5-2x^2)^{-7/4}(-4x)$$

$$= 9\pi x\,(5-2x^2)^{-7/4}$$

$$= \frac{9\pi x}{(5-2x^2)^{7/4}}$$

(33)

$$y = \frac{(4x+3)^{1/2}}{2x}$$

$$y' = \frac{2x\left[\frac{1}{2}(4x+3)^{-1/2}(4)\right] - (4x+3)^{1/2}(2)}{4x^2}$$

$$= \frac{\left[4x(4x+3)^{-1/2} - 2(4x+3)^{1/2}\right]}{4x^2} \qquad \frac{(4x+3)^{1/2}}{(4x+3)^{1/2}}$$

$$= \frac{4x - 2(4x+3)}{4x^2(4x+3)^{1/2}}$$

$$= \frac{4x - 8x - 6}{4x^2(4x+3)^{1/2}}$$

$$= \frac{-4x-6}{4x^2(4x+3)^{-1/2}} \quad \text{or} \quad \frac{-2x-3}{2x^2(4x+3)^{1/2}}$$

(35) $(2x-3y)^3 = x^2 - y$

Take $\frac{d}{dx}$: $3(2x-3y)^2\left(2 - 3\frac{dy}{dx}\right) = 2x - \frac{dy}{dx}$

$$6(2x-3y)^2 - 9(2x-3y)^2\frac{dy}{dx} = 2x - \frac{dy}{dx}$$

$$\frac{dy}{dx} - 9(2x-3y)^2\frac{dy}{dx} = 2x - 6(2x-3y)^2$$

$$\left[1 - 9(2x-3y)^2\right]\frac{dy}{dx} = 2x - 6(2x-3y)^2$$

$$\frac{dy}{dx} = \frac{2x - 6(2x-3y)^2}{1 - 9(2x-3y)^2}$$

(39) $y = 2x(12x+7)^{1/2}$

$y' = 2x\left[\frac{1}{2}(12x+7)^{-1/2}(12)\right] + (12x+7)^{1/2}(2)$

$y'\big|_{x=1.5} = 3\left[6(25)^{-1/2}\right] + 25^{1/2}(2)$

$= \frac{18}{5} + 10$

$= 13.6$

(41) $y = 3x^4 - x^{-1}$

$y' = 12x^3 + x^{-2}$

$y'' = 36x^2 - 2x^{-3}$

or $y'' = 36x^2 - \frac{2}{x^3}$

or $y'' = \frac{1}{x^3}(36x^5 - 2)$

or $y'' = \frac{2(18x^5-1)}{x^3}$

59

$$s = (1+8t)^{1/2}$$

$$v = \tfrac{1}{2}(1+8t)^{-1/2}(8)$$

$$= 4(1+8t)^{-1/2}$$

$$a = -2(1+8t)^{-3/2}(8)$$

$$= -16(1+8t)^{-3/2}$$

Could be written as :

$$v = \frac{4}{(1+8t)^{1/2}} \quad \text{and} \quad a = \frac{-16}{(1+8t)^{3/2}}$$