

$$\begin{aligned} \textcircled{3} \quad x^2 y \\ \text{Take } \frac{d}{dx} : \quad x^2 \frac{dy}{dx} + y(2x) \\ = x^2 \frac{dy}{dx} + 2xy \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad 3x + 2y = 5 \\ \text{Take } \frac{d}{dx} : \quad 3 + 2 \frac{dy}{dx} = 0 \\ 2 \frac{dy}{dx} = -3 \\ \frac{dy}{dx} = -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} \textcircled{11} \quad x^2 - 4y^2 - 9 = 0 \\ \text{Take } \frac{d}{dx} : \quad 2x - 8y \frac{dy}{dx} = 0 \\ -8y \frac{dy}{dx} = -2x \\ \frac{dy}{dx} = \frac{-2x}{-8y} \\ \frac{dy}{dx} = \frac{x}{4y} \end{aligned}$$

$$(15) \quad y^2 + y = x^2 - 4$$

$$\text{Take } \frac{d}{dx}: \quad 2y \frac{dy}{dx} + \frac{dy}{dx} = 2x$$

$$[2y + 1] \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2y + 1}$$

$$(19) \quad x^2 = \frac{x-y}{x+y}$$

$$\text{Take } \frac{d}{dx}: \quad 2x = \frac{(x+y)(1 - \frac{dy}{dx}) - (x-y)(1 + \frac{dy}{dx})}{(x+y)^2}$$

$$2x(x+y)^2 = (x+y)(1 - \frac{dy}{dx}) - \overset{+(-x+y)}{\cancel{(x-y)}}(1 + \frac{dy}{dx})$$

$$2x(x+y)^2 = x - x \frac{dy}{dx} + y - y \frac{dy}{dx} \\ - x - x \frac{dy}{dx} + y + y \frac{dy}{dx}$$

$$x \frac{dy}{dx} + y \frac{dy}{dx} + x \frac{dy}{dx} - y \frac{dy}{dx} = 2y - 2x(x+y)^2$$

$$2x \frac{dy}{dx} = 2y - 2x(x+y)^2$$

$$\frac{dy}{dx} = \frac{2y - 2x(x+y)^2}{2x}$$

→

$$\text{or } \frac{dy}{dx} = \frac{y - x(x+y)^2}{x}$$

$$\text{or } \frac{dy}{dx} = \frac{y - x(x^2 + 2xy + y^2)}{x}$$

$$\frac{dy}{dx} = \frac{y - x^3 - 2x^2y - xy^2}{x}$$

$$(23) \quad (2y-x)^4 + x^2 = y+3$$

$$\text{Take } \frac{d}{dx} : \quad 4(2y-x)^3 (2 \frac{dy}{dx} - 1) + 2x = \frac{dy}{dx}$$

$$8(2y-x)^3 \frac{dy}{dx} - 4(2y-x)^3 + 2x = \frac{dy}{dx}$$

$$8(2y-x)^3 \frac{dy}{dx} - \frac{dy}{dx} = 4(2y-x)^3 - 2x$$

$$[8(2y-x)^3 - 1] \frac{dy}{dx} = 4(2y-x)^3 - 2x$$

$$\frac{dy}{dx} = \frac{4(2y-x)^3 - 2x}{8(2y-x)^3 - 1}$$

$$(27) \quad 3x^3 y^2 - 2y^3 = -4$$

$$\text{Take } \frac{dy}{dx} = 3x^3 (2y \frac{dy}{dx}) + y^2 (9x^2) - 6y^2 \frac{dy}{dx} = 0$$

$$6x^3 y \frac{dy}{dx} + 9x^2 y^2 - 6y^2 \frac{dy}{dx} = 0$$

$$6x^3 y \frac{dy}{dx} - 6y^2 \frac{dy}{dx} = -9x^2 y^2$$

$$[6x^3 y - 6y^2] \frac{dy}{dx} = -9x^2 y^2$$

$$\frac{dy}{dx} = \frac{-9x^2 y^2}{6x^3 y - 6y^2}$$

$$\frac{dy}{dx} \Big|_{(x,y)=(1,2)} = \frac{-36}{12-24}$$

$$= \frac{-36}{-12}$$

$$= 3$$

$$(29) \quad 5y^4 + 7 = x^4 - 3y$$

$$\text{Take } \frac{d}{dx}: 20y^3 \frac{dy}{dx} = 4x^3 - 3 \frac{dy}{dx}$$

$$20y^3 \frac{dy}{dx} + 3 \frac{dy}{dx} = 4x^3$$

$$[20y^3 + 3] \frac{dy}{dx} = 4x^3$$

$$\frac{dy}{dx} = \frac{4x^3}{20y^3 + 3}$$

$$\frac{dy}{dx} \Big|_{(x,y)=(3,-2)} = \frac{108}{-157}$$

$$= -\frac{108}{157}$$

$$(31) \quad xy^2 + 3x^2 - y^2 + 15 = 0$$

$$\text{Take } \frac{d}{dx} : \quad x \left(2y \frac{dy}{dx} \right) + y^2 (1) + 6x - 2y \frac{dy}{dx} = 0$$

$$2xy \frac{dy}{dx} - 2y \frac{dy}{dx} = -y^2 - 6x$$

$$\left[2xy \frac{dy}{dx} - 2y \right] \frac{dy}{dx} = -y^2 - 6x$$

$$\frac{dy}{dx} = \frac{-y^2 - 6x}{2xy - 2y}$$

$$\frac{dy}{dx} \Big|_{(x,y)=(-1,3)} = \frac{-9+6}{-6-6}$$

$$= \frac{-3}{-12}$$

$$= \frac{1}{4}$$

(33)

$$x^2 + y^2 = 4x$$

Take $\frac{d}{dx}$: $2x + 2y \frac{dy}{dx} = 4$

$$2y \frac{dy}{dx} = 4 - 2x$$

$$\frac{dy}{dx} = \frac{4 - 2x}{2y}$$

Horizontal tangent line $\Rightarrow \frac{dy}{dx} = 0$

Set $\frac{dy}{dx} = 0$: $\frac{4 - 2x}{2y} = 0$

$$4 - 2x = 0$$

$$4 = 2x$$

$$x = 2$$

To find the y -value(s), sub $x = 2$ into the original equation.

$$x = 2 \rightarrow x^2 + y^2 = 4x$$

$$4 + y^2 = 8$$

$$y^2 = 4$$

$$y = \pm 2$$

The points are $(x, y) = (2, 2)$ and $(2, -2)$.