(5)

$$y = \sqrt{x}$$

$$y = x^{1/2}$$

$$y' = \frac{1}{2} x^{-1/2} \quad \text{or} \quad y' = \frac{1}{2\sqrt{x}}$$

(9)

$$y = \frac{3}{\sqrt[3]{x}}$$

$$y = 3x^{-1/3}$$

$$y' = -x^{-4/3} \quad \text{or} \quad y' = \frac{-1}{x^{4/3}}$$

(13)

$$y = (x^2+1)^5$$

$$y' = 5(x^2+1)^4(2x)$$

$$y' = 10x(x^2+1)^4$$

(17) $y = (2x^3 - 3)^{1/3}$

$y' = \frac{1}{3}(2x^3 - 3)^{-2/3}(6x^2)$

$y' = 2x^2(2x^3 - 3)^{-2/3}$ or $y' = \dfrac{2x^2}{(2x^3 - 3)^{2/3}}$

(21) $y = 4(2x^4 - 5)^{0.75}$

$y' = 3(2x^4 - 5)^{-0.25}(8x^3)$

$y' = 24x^3(2x^4 - 5)^{-0.25}$

or $y' = \dfrac{24x^3}{(2x^4 - 5)^{0.25}}$

or $y' = \dfrac{24x^3}{(2x^4 - 5)^{1/4}}$

or $y' = \dfrac{24x^3}{\sqrt[4]{2x^4 - 5}}$

(25)

$$u = v(8v+5)^{1/2}$$

$$u' = v\,\tfrac{1}{2}(8v+5)^{-1/2}(8) + (8v+5)^{1/2}(1)$$

$$u' = \frac{4v}{\sqrt{8v+5}} + \sqrt{8v+5}\cdot\frac{\sqrt{8v+5}}{\sqrt{8v+5}}$$

$$u' = \frac{4v + (8v+5)}{\sqrt{8v+5}}$$

$$u' = \frac{12v+5}{\sqrt{8v+5}}$$

29 $y = \dfrac{[2x(x+2)^{1/2}]}{x+4}$ $\leftarrow u$ $\leftarrow v$

$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

$$y' = \frac{(x+4)\frac{d}{dx}[2x(x+2)^{1/2}] - 2x(x+2)^{1/2}(1)}{(x+4)^2}$$

$$y' = \frac{(x+4)[2x\,\frac{1}{2}(x+2)^{-1/2}(1) + (x+2)^{1/2}(2)] - 2x(x+2)^{1/2}}{(x+4)^2}$$

$$y' = \frac{(x+4)\,x\,(x+2)^{-1/2} + 2(x+4)(x+2)^{1/2} - 2x(x+2)^{1/2}}{(x+4)^2}$$

$$\boxed{\text{Multiply by } \frac{(x+2)^{1/2}}{(x+2)^{1/2}}}$$

$$y' = \frac{(x+4)x + 2(x+4)(x+2) - 2x(x+2)}{(x+4)^2(x+2)^{1/2}}$$

$$y' = \frac{x^2 + 4x + (2x+8)(x+2) - 2x^2 - 4x}{(x+4)^2(x+2)^{1/2}}$$

$$y' = \frac{-x^2 + 2x^2 + 4x + 8x + 16}{(x+4)^2(x+2)^{1/2}}$$

$$y' = \frac{x^2 + 12x + 16}{(x+4)^2(x+2)^{1/2}}$$

33

$$y = (3x+4)^{1/2}$$

$$y' = \frac{1}{2}(3x+4)^{-1/2}(3)$$

$$= \frac{3}{2\sqrt{3x+4}}$$

$$y'\Big|_{x=7} = \frac{3}{2\sqrt{25}}$$

$$= \frac{3}{10} \quad \text{or} \quad 0.3$$

37

$$y = x(x^{1/2})$$

$$y' = x\left(\frac{1}{2}x^{-1/2}\right) + x^{1/2}(1)$$

$$= \frac{1}{2}x^{1/2} + x^{1/2}$$

$$= \frac{3}{2}x^{1/2}$$

39 a)

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

$$y = \frac{1}{x^3}$$

$$y' = \frac{x^3(0) - 1(3x^2)}{(x^3)^2}$$

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^4}$$

b)

$$y = x^{-3}$$

$$y' = -3x^{-4}$$

$$= \frac{-3}{x^4}$$

(41) $$y = \frac{x^2}{(x^2+1)^{1/2}}$$

$$y' = \frac{(x^2+1)^{1/2}(2x) - x^2\left[\frac{1}{2}(x^2+1)^{-1/2}(2x)\right]}{x^2+1}$$

$$= \frac{2x(x^2+1)^{1/2} - x^3(x^2+1)^{-1/2}}{x^2+1}$$

$$\boxed{\text{Multiply by } \frac{(x^2+1)^{1/2}}{(x^2+1)^{1/2}}}$$

$$= \frac{2x(x^2+1) - x^3}{(x^2+1)^{3/2}}$$

$$= \frac{2x^3 + 2x - x^3}{(x^2+1)^{3/2}}$$

$$= \frac{x^3 + 2x}{(x^2+1)^{3/2}}$$

Set $y' = 0$: $$\frac{x^3+2x}{(x^2+1)^{3/2}} = 0$$

From Wolfram Alpha

$$y = \frac{x^2}{\sqrt{x^2+1}}$$

"sharp corner" at $x=0$

$$x^3 + 2x = 0$$

$$x(x^2+2) = 0$$

$x = 0$

$x^2 + 2 = 0$ no solution

$$\boxed{x=0}$$

(49) $s = (8t - t^2)^{2/3}$

$$v = \frac{2}{3}(8t - t^2)^{-1/3}(8 - 2t)$$

$$= \frac{2(8-2t)}{3 \cdot \sqrt[3]{8t - t^2}}$$

$$v(6.25) = \frac{-9}{3 \cdot \sqrt[3]{10.9375}}$$

$$\approx -1.35 \qquad \text{cm/s}$$

(53) Note: a and k are positive constants

$$v = k\left(\frac{l}{a} + \frac{a}{l}\right)^{1/2}$$

$$v = k\left(\frac{l}{a} + a l^{-1}\right)^{1/2}$$

$$\frac{dv}{dl} = k \, \frac{1}{2}\left(\frac{l}{a} + a l^{-1}\right)^{-1/2}\left(\frac{1}{a} - a l^{-2}\right)$$

$$= \frac{k\left(\frac{1}{a} - a l^{-2}\right)}{2\left(\frac{l}{a} + a l^{-1}\right)^{1/2}}$$

Set $\frac{dv}{dl} = 0$: $\quad \frac{k\left(\frac{1}{a} - a l^{-2}\right)}{2\left(\frac{l}{a} + a l^{-1}\right)^{1/2}} = 0$

$$k\left(\frac{1}{a} - a l^{-2}\right) = 0 \qquad \rightarrow$$

$$\frac{1}{a} - a l^{-2} = 0$$

$$\frac{1}{a} - \frac{a}{l^2} = 0$$

$$\frac{1}{a} = \frac{a}{l^2}$$

$$l^2 = a^2$$

$$l = \pm a$$

But l is a length so $l > 0$

$$\boxed{l = a}$$