

Section 16.5

$$\textcircled{3} \quad \begin{array}{cc|c} & x & y \\ \hline 1 & 2 & 4 \\ 3 & -1 & 5 \end{array}$$

$$R_2 - 3R_1 \quad \begin{array}{cc|c} 1 & 2 & 4 \\ \hline 0 & -7 & -7 \end{array}$$

$$\frac{R_2}{(-7)} \quad \begin{array}{cc|c} 1 & 2 & 4 \\ \hline 0 & 1 & 1 \end{array}$$

$$R_1 - 2R_2 \quad \begin{array}{cc|c} 1 & 0 & 2 \\ \hline 0 & 1 & 1 \end{array}$$

$$(x, y) = (2, 1)$$

⑦ Notice that the second equation must be reordered:

$$-4x + 6y - 2z = 9$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 2 & -3 & 1 & 4 \\ -4 & 6 & -2 & 9 \end{array}$$

$$\frac{R_1}{2} \begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 2 \\ \hline -4 & 6 & -2 & 9 \end{array}$$

$$R_2 + 4R_1 \begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 2 \\ \hline 0 & 0 & 0 & 17 \end{array}$$

$$0 = 17$$

Impossible

The system has no solution.

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$$\left[ \begin{array}{ccc|c} 1 & 3 & 3 & -3 \\ 2 & 2 & 1 & -5 \\ -2 & -1 & 4 & 6 \end{array} \right]$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 + 2R_1 \end{array} \left[ \begin{array}{ccc|c} 1 & 3 & 3 & -3 \\ 0 & -4 & -5 & 1 \\ 0 & 5 & 10 & 0 \end{array} \right]$$

$$\frac{R_2}{-4} \left[ \begin{array}{ccc|c} 1 & 3 & 3 & -3 \\ 0 & 1 & \frac{5}{4} & -\frac{1}{4} \\ 0 & 5 & 10 & 0 \end{array} \right]$$

$$\begin{array}{l} R_1 - 3R_2 \\ R_3 - 5R_2 \end{array} \left[ \begin{array}{ccc|c} 1 & 0 & -\frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & \frac{5}{4} & -\frac{1}{4} \\ 0 & 0 & \frac{15}{4} & \frac{5}{4} \end{array} \right]$$

$$\frac{4}{15} \times R_3 \left[ \begin{array}{ccc|c} 1 & 0 & -\frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & \frac{5}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{3} \end{array} \right]$$

$$\begin{array}{l} R_1 + \frac{3}{4}R_3 \\ R_2 - \frac{5}{4}R_3 \end{array} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{3} \end{array} \right]$$

$$(x, y, z) = \left(-2, -\frac{2}{3}, \frac{1}{3}\right)$$

(11)

$$\begin{bmatrix} w & x & y & z & \\ 1 & 2 & -1 & 3 & 12 \\ 2 & 0 & -2 & -1 & 3 \\ 0 & 3 & -1 & -1 & -1 \\ -1 & 2 & 1 & 2 & 3 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_4 + R_1 \end{array} \begin{bmatrix} 1 & 2 & -1 & 3 & 12 \\ 0 & -4 & 0 & -7 & -21 \\ 0 & 3 & -1 & -1 & -1 \\ 0 & 4 & 0 & 5 & 15 \end{bmatrix}$$

$$\frac{R_2}{(-4)} \begin{bmatrix} 1 & 2 & -1 & 3 & 12 \\ 0 & 1 & 0 & 7/4 & 21/4 \\ 0 & 3 & -1 & -1 & -1 \\ 0 & 4 & 0 & 5 & 15 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_3 - 3R_2 \\ R_4 - 4R_2 \end{array} \begin{bmatrix} 1 & 0 & -1 & -1/2 & 3/2 \\ 0 & 1 & 0 & 7/4 & 21/4 \\ 0 & 0 & -1 & -25/4 & -67/4 \\ 0 & 0 & 0 & -2 & -6 \end{bmatrix}$$

$$\frac{R_3}{(-1)} \begin{bmatrix} 1 & 0 & -1 & -1/2 & 3/2 \\ 0 & 1 & 0 & 7/4 & 21/4 \\ 0 & 0 & 1 & 25/4 & 67/4 \\ 0 & 0 & 0 & -2 & -6 \end{bmatrix}$$

$$R_1 + R_3 \begin{bmatrix} 1 & 0 & 0 & 23/4 & 73/4 \\ 0 & 1 & 0 & 7/4 & 21/4 \\ 0 & 0 & 1 & 25/4 & 67/4 \\ 0 & 0 & 0 & -2 & -6 \end{bmatrix}$$

$$\frac{R_4}{(-2)} \begin{bmatrix} 1 & 0 & 0 & 23/4 & 73/4 \\ 0 & 1 & 0 & 7/4 & 21/4 \\ 0 & 0 & 1 & 25/4 & 67/4 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 23R_4 \\ R_2 - 7/4 R_4 \\ R_3 - 25/4 R_4 \end{array} \begin{bmatrix} w & x & y & z & \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

$$(w, x, y, z) = (1, 0, -2, 3)$$

(13)

$$\left[ \begin{array}{ccc|c} 1 & -4 & 1 & 2 \\ 3 & -1 & 4 & -4 \end{array} \right]$$

$$R_2 - 3R_1 \left[ \begin{array}{ccc|c} 1 & -4 & 1 & 2 \\ 0 & 11 & 1 & -10 \end{array} \right]$$

$$\frac{R_2}{11} \left[ \begin{array}{ccc|c} 1 & -4 & 1 & 2 \\ 0 & 1 & \frac{1}{11} & -\frac{10}{11} \end{array} \right]$$

$$R_1 + 4R_2 \left[ \begin{array}{ccc|c} 1 & 0 & \frac{15}{11} & -\frac{18}{11} \\ 0 & 1 & \frac{1}{11} & -\frac{10}{11} \end{array} \right]$$

$$\uparrow \\ z = t$$

$$x + \frac{15}{11}z = \frac{-18}{11} \rightarrow x = \frac{-18}{11} - \frac{15}{11}t$$

$$y + \frac{1}{11}z = \frac{-10}{11} \rightarrow y = \frac{-10}{11} - \frac{1}{11}t$$

$$(x, y, z) = \left( \frac{-18}{11} - \frac{15}{11}t, \frac{-10}{11} - \frac{1}{11}t, t \right)$$

Two particular solutions:

$$(t=0) \quad (x, y, z) = \left( \frac{-18}{11}, \frac{-10}{11}, 0 \right)$$

$$(t=1) \quad (x, y, z) = (-3, -1, 1)$$

(15)

$$\begin{bmatrix} 2 & -1 & 1 & | & 5 \\ 3 & 2 & -2 & | & 4 \\ 5 & 8 & -8 & | & 5 \end{bmatrix}$$

$$\frac{R_1}{2} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{5}{2} \\ 3 & 2 & -2 & | & 4 \\ 5 & 8 & -8 & | & 5 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 5R_1 \end{array} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{5}{2} \\ 0 & \frac{7}{2} & -\frac{7}{2} & | & -\frac{7}{2} \\ 0 & \frac{21}{2} & -\frac{21}{2} & | & -\frac{15}{2} \end{bmatrix}$$

$$\frac{2}{7} \times R_2 \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{5}{2} \\ 0 & 1 & -1 & | & -1 \\ 0 & \frac{21}{2} & -\frac{21}{2} & | & -\frac{15}{2} \end{bmatrix}$$

$$\begin{array}{l} R_1 + \frac{1}{2}R_2 \\ R_3 - \frac{21}{2}R_2 \end{array} \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & -1 & | & -1 \\ 0 & 0 & 0 & | & 3 \end{bmatrix}$$

$$0 = 3$$

Impossible.

The system has no solution.

(17)

$$\left[ \begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 2 & -6 & -3 & 10 \\ 4 & -9 & 3 & 4 \end{array} \right]$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \left[ \begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 0 & -12 & -5 & 2 \\ 0 & -21 & -1 & -12 \end{array} \right]$$

$$\frac{R_2}{(-12)} \left[ \begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 0 & 1 & 5/12 & -1/6 \\ 0 & -21 & -1 & -12 \end{array} \right]$$

$$\begin{array}{l} R_1 - 3R_2 \\ R_3 + 21R_2 \end{array} \left[ \begin{array}{ccc|c} 1 & 0 & -1/4 & 9/2 \\ 0 & 1 & 5/12 & -1/6 \\ 0 & 0 & 31/4 & -31/2 \end{array} \right]$$

$$\frac{4}{31} \times R_3 \left[ \begin{array}{ccc|c} 1 & 0 & -1/4 & 9/2 \\ 0 & 1 & 5/12 & -1/6 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\begin{array}{l} R_1 + \frac{1}{4}R_3 \\ R_2 - \frac{5}{12}R_3 \end{array} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2/3 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$(x, y, z) = (4, \frac{2}{3}, -2)$$

(19)

$$\left[ \begin{array}{cc|c} 2 & -4 & 7 \\ 3 & 5 & -6 \\ 9 & -7 & 15 \end{array} \right]$$

$$\frac{R_1}{2} \left[ \begin{array}{cc|c} 1 & -2 & 7/2 \\ 3 & 5 & -6 \\ 9 & -7 & 15 \end{array} \right]$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 9R_1 \end{array} \left[ \begin{array}{cc|c} 1 & -2 & 7/2 \\ 0 & 11 & -33/2 \\ 0 & 11 & -33/2 \end{array} \right]$$

$$\frac{R_2}{11} \left[ \begin{array}{cc|c} 1 & -2 & 7/2 \\ 0 & 1 & -3/2 \\ 0 & 11 & -33/2 \end{array} \right]$$

$$\begin{array}{l} R_1 + 2R_2 \\ R_3 - 11R_2 \end{array} \left[ \begin{array}{cc|c} 1 & 0 & 1/2 \\ 0 & 1 & -3/2 \\ 0 & 0 & 0 \end{array} \right] \leftarrow 0=0 \text{ NO INFO}$$

$$(x, y) = \left( \frac{1}{2}, -\frac{3}{2} \right)$$



(25) Notice the second equation needs reordering:

$$3s + 6t - 9u = 6$$

$$\begin{array}{ccc|c} s & t & u & \\ \hline 1 & 2 & -3 & 2 \\ 3 & 6 & -9 & 6 \\ 7 & 14 & -21 & 13 \end{array}$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 7R_1 \end{array} \begin{array}{ccc|c} 1 & 2 & -3 & 2 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{array}$$

$$0 = -1$$

Impossible

The system has no solution.

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$$\begin{array}{c} r \quad s \quad t \quad u \\ \left[ \begin{array}{cccc|c} 1 & -1 & -3 & -1 & 1 \\ 2 & 4 & 0 & -2 & 2 \\ 1 & 5 & 3 & -1 & 1 \\ 3 & 4 & -2 & 0 & 0 \\ 1 & 0 & 2 & -3 & 3 \end{array} \right] \end{array}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - R_1 \\ R_4 - 3R_1 \\ R_5 - R_1 \end{array} \left[ \begin{array}{cccc|c} 1 & -1 & 3 & -1 & 1 \\ 0 & 6 & 6 & 0 & 0 \\ 0 & 6 & 6 & 0 & 0 \\ 0 & 7 & 7 & 3 & -3 \\ 0 & 1 & 5 & -2 & 2 \end{array} \right]$$

$$\frac{R_2}{6} \left[ \begin{array}{cccc|c} 1 & -1 & 3 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 6 & 6 & 0 & 0 \\ 0 & 7 & 7 & 3 & -3 \\ 0 & 1 & 5 & -2 & 2 \end{array} \right]$$

$$\begin{array}{l} R_1 + R_2 \\ R_3 - 6R_2 \\ R_4 - 7R_2 \\ R_5 - R_2 \end{array} \left[ \begin{array}{cccc|c} 1 & 0 & 4 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 \\ 0 & 0 & 4 & -2 & 2 \end{array} \right]$$

$$R_3 \leftrightarrow R_5 \left[ \begin{array}{cccc|c} 1 & 0 & 4 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 4 & -2 & 2 \\ 0 & 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\frac{R_3}{4} \left[ \begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} R_1 - 4R_3 \\ R_2 - R_3 \end{array} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\frac{R_4}{3} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} R_1 - R_4 \\ R_2 - \frac{1}{2}R_4 \\ R_3 + \frac{1}{2}R_4 \end{array} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(Bottom row is  $0=0$ , which gives no info.)

$$(r, s, t, u) = (0, 0, 0, -1)$$