Exam Review 15-23

15. a) $\int 2x^2\sqrt{1-4x^3}\,dx$

$= 2\int x^2(1-4x^3)^{1/2}\,dx$

let $u = 1-4x^3$

$du = -12x^2\,dx$

$-\frac{1}{12}\,du = x^2\,dx$

$= -\frac{2}{12}\int u^{1/2}\,du$

$= -\frac{1}{6}\cdot\frac{2}{3}u^{3/2} + C$

$= -\frac{1}{9}(1-4x^3)^{3/2} + C$

b) $\int \frac{2x-1}{x^3}\,dx$

$= \int\left(\frac{2x}{x^3} - \frac{1}{x^3}\right)dx$

$= \int\left(2x^{-2} - x^{-3}\right)dx$

$= 2\left(\frac{1}{-1}x^{-1}\right) - \left(\frac{1}{-2}\right)x^{-2} + C$

$= -\frac{2}{x} + \frac{1}{2x^2} + C$

or $\frac{1-4x}{2x^2} + C$

c) $\displaystyle\int_0^1 \frac{3x^4}{(1+7x^5)^2}\, dx$

let $u = 1+7x^5$
$du = 35x^4\, dx$
$\frac{1}{35}\, du = x^4\, dx$

$$= \frac{3}{35} \int_{x=0}^{x=1} u^{-2}\, du$$

$$= \frac{3}{35} \left(\frac{1}{-1} u^{-1} \right) \Bigg|_{x=0}^{x=1}$$

$$= -\frac{3}{35} \left(\frac{1}{1+7x^5} \right) \Bigg|_0^1$$

$$= \frac{3}{40} \quad \text{or} \quad 0.075$$

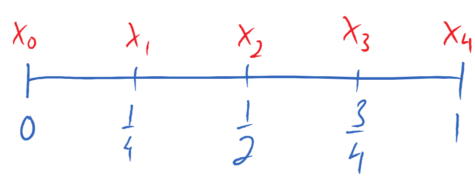
d) $\displaystyle\int (3t^2+1)^2\, dt$

$$= \int (9t^4 + 6t^2 + 1)\, dt$$

$$= \frac{9}{5} t^5 + 2t^3 + t + C$$

16. $\displaystyle\int_0^1 \sin x^3\, dx \qquad n = 4$

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$$



radians!

$$\int_0^1 \sin x^3\, dx \approx \frac{\frac{1}{4}}{3} \left[\sin 0^3 + 4 \sin\left(\tfrac{1}{4}\right)^3 + 2 \sin\left(\tfrac{1}{2}\right)^3 + 4 \sin\left(\tfrac{3}{4}\right)^3 + \sin 1^3 \right]$$

$$\approx 0.2326$$

17. $a = 12t$

Plan: find v using $v = \int a\,dt$ then find s using $s = \int v\,dt$

$$v = \int a\,dt$$

$$= \int 12t\,dt$$

$$v = 6t^2 + C_1$$

solve for C_1 using $v = 5$ when $t = 0$

$$5 = 6(0)^2 + C_1$$

$$C_1 = 5$$

so $v = 6t^2 + 5$

$$s = \int v\,dt$$

$$= \int (6t^2 + 5)\,dt$$

$$s = 2t^3 + 5t + C_2$$

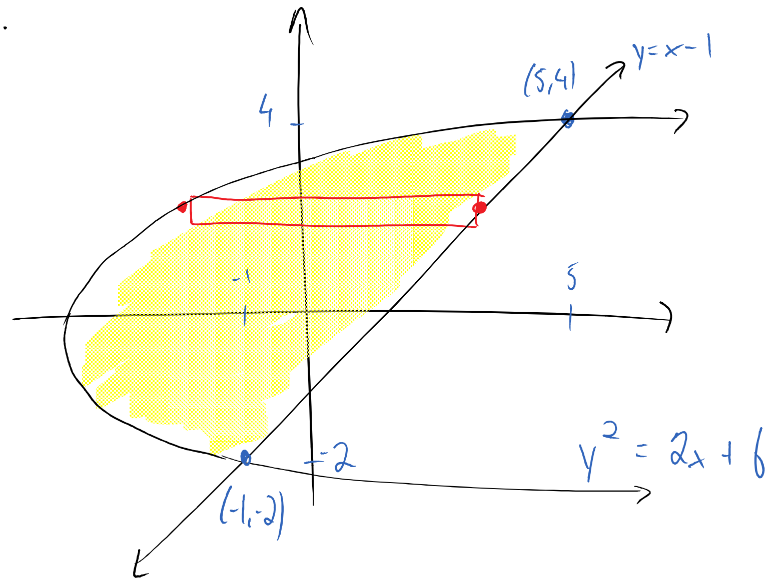
solve for C_2 using $s = 0$ when $t = 0$

$$0 = 2(0)^3 + 5(0) + C_2$$

$$C_2 = 0$$

so $s = 2t^3 + 5t$ m

18.



intersection points:

$$y = y$$
$$y^2 = y^2$$
$$2x+6 = (x-1)^2$$
$$2x+6 = x^2 - 2x + 1$$
$$0 = x^2 - 4x - 5$$
$$0 = (x-5)(x+1)$$

$$x = 5, -1$$

$$x=5 \Rightarrow y = 5-1 = 4 \Rightarrow (5,4)$$
$$x=-1 \Rightarrow y = -1-1 = -2 \Rightarrow (-1,-2)$$

$$dA = \text{area of rectangle}$$
$$= (\text{width})(\text{height})$$
$$= (\text{right} - \text{left})\, dy$$
$$= \left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right] dy$$
$$= \left[y + 4 - \tfrac{1}{2}y^2\right] dy$$

right

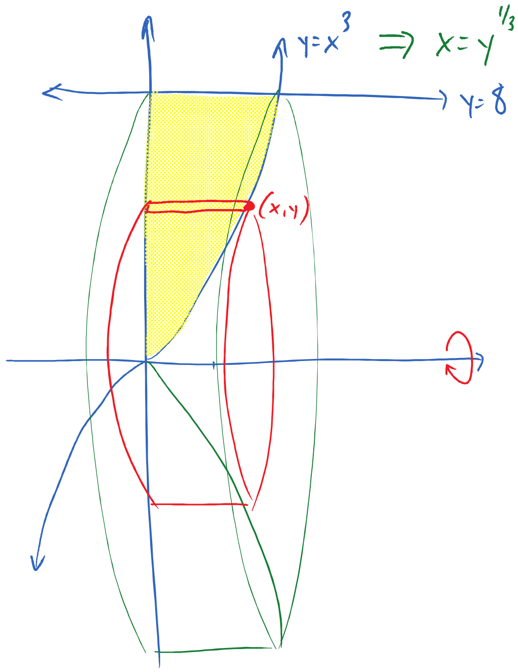
$$y = x-1$$
$$x = y+1$$

left

$$y^2 = 2x+6$$
$$y^2 - 6 = 2x$$
$$x = \tfrac{1}{2}y^2 - 3$$

$$A = \int_A dA = \int_{-2}^{4} \left(y + 4 - \tfrac{1}{2}y^2\right) dy$$
$$= \left[\tfrac{1}{2}y^2 + 4y - \tfrac{1}{6}y^3\right]_{-2}^{4}$$
$$= \left[\tfrac{1}{2}(4)^2 + 4(4) - \tfrac{1}{6}(4)^3\right] - \left[\tfrac{1}{2}(-2)^2 + 4(-2) - \tfrac{1}{6}(-2)^3\right]$$
$$= 18$$

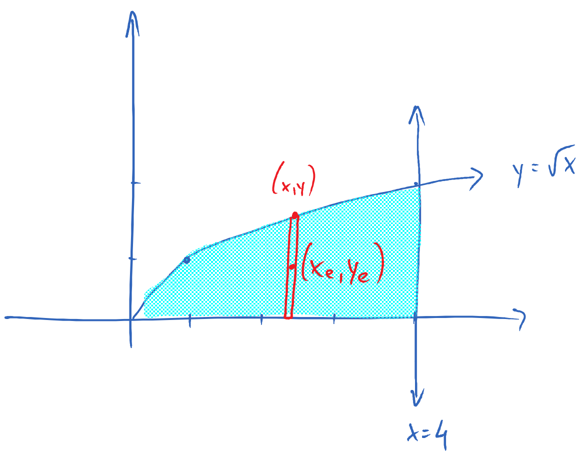
19.



shell: $dV = 2\pi r h\, dt$
$= 2\pi y x\, dy$
$= 2\pi y\, y^{1/3}\, dy$
$= 2\pi y^{4/3}\, dy$

$V = \int_V dV$
$= 2\pi \int_0^8 y^{4/3}\, dy$
$= 2\pi \frac{3}{7} y^{7/3} \Big|_0^8$
$= \frac{768\pi}{7}$

20.



$x_e = x$

$y_e = \text{midpoint} = \frac{y}{2} = \frac{x^{1/2}}{2}$

dA = area of rectangle
= (height)(width)
$= y\, dx$

$dA = x^{1/2}\, dx$

$A = \int_A dA$
$= \int_0^4 x^{1/2}\, dx$
$= \frac{2}{3} x^{3/2} \Big|_0^4$

$A = \frac{16}{3}$

$$\bar{x} = \frac{1}{A}\int_A x_e \, dA$$

$$= \frac{3}{16}\int_0^4 \underbrace{x}_{x_e} \underbrace{x^{1/2}\,dx}_{dA}$$

$$= \frac{3}{16}\int_0^4 x^{3/2}\,dx$$

$$= \frac{3}{16}\cdot\frac{2}{5}x^{5/2}\Big|_0^4$$

$$= \frac{12}{5} \quad \text{or} \quad 2.4$$

$$\bar{y} = \frac{1}{A}\int_A y_e \, dA$$

$$= \frac{3}{16}\int_0^4 \underbrace{\frac{x^{1/2}}{2}}_{y_e} \underbrace{x^{1/2}\,dx}_{dA}$$

$$= \frac{3}{32}\int_0^4 x\,dx$$

$$= \frac{3}{32}\cdot\frac{1}{2}x^2\Big|_0^4$$

$$= \frac{3}{4} \quad \text{or} \quad 0.75$$

$$(\bar{x}, \bar{y}) = \left(\frac{12}{5}, \frac{3}{4}\right)$$

21. $s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$y = 0.04x^{3/2}$

$\frac{dy}{dx} = 0.06x^{1/2}$

$\left(\frac{dy}{dx}\right)^2 = \left(0.06x^{1/2}\right)^2 = 0.0036x$

$s = \int_0^{100} \sqrt{1 + 0.0036x}\, dx$

let $u = 1 + 0.0036x$
$du = 0.0036\, dx$
$\frac{1}{0.0036}\, du = dx$

$= \frac{1}{0.0036} \int_{x=0}^{x=100} u^{1/2}\, du$

$= \frac{1}{0.0036} \cdot \frac{2}{3} u^{3/2} \Big|_{x=0}^{x=100}$

$= \frac{2}{0.0108} (1 + 0.0036x)^{3/2} \Big|_0^{100}$

$= \frac{1}{0.0108} \left[(1 + 0.0036(100))^{3/2} - 1^{3/2} \right]$

$= 108.52$ m

22. a)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & -4 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -1 & 2 & -2 & 1 & 0 \\ 0 & -6 & 10 & -3 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -2 & 2 & -1 & 0 \\ 0 & -6 & 10 & -3 & 0 & 1 \end{array}\right] \xrightarrow[R_3 + 6R_2]{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & -2 & 2 & -1 & 0 \\ 0 & 0 & -2 & 9 & -6 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & 0 & -7 & 5 & -1 \\ 0 & 0 & -2 & 9 & -6 & 1 \end{array}\right] \xrightarrow{-\frac{1}{2}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & 0 & -7 & 5 & -1 \\ 0 & 0 & 1 & -9/2 & 3 & -1/2 \end{array}\right]$$

$$\xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -1 & 1/2 \\ 0 & 1 & 0 & -7 & 5 & -1 \\ 0 & 0 & 1 & -9/2 & 3 & -1/2 \end{array}\right]$$

A^{-1}

b) $AX = C$

$$X = A^{-1}C = \begin{bmatrix} 3/2 & -1 & 1/2 \\ -7 & 5 & -1 \\ -9/2 & 3 & -1/2 \end{bmatrix} \begin{bmatrix} -11 \\ -14 \\ 9 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

so $x = 2,\ y = -2,\ z = 3$

(23) Solve using Gauss-Jordan Elimination:

$$\begin{cases} x + 2y - 3z = -11 \\ 2x + 3y - 4z = -14 \\ 3x + z = 9 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -11 \\ 2 & 3 & -4 & -14 \\ 3 & 0 & 1 & 9 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -11 \\ 0 & -1 & 2 & 8 \\ 0 & -6 & 10 & 42 \end{array}\right]$$

$\frac{R_2}{-1}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -11 \\ 0 & 1 & -2 & -8 \\ 0 & -6 & 10 & 42 \end{array}\right]$$

$R_1 - 2R_2$
$R_3 + 6R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & -2 & -8 \\ 0 & 0 & -2 & -6 \end{array}\right]$$

$\frac{R_3}{-2}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & -2 & -8 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_1 - R_3$
$R_2 + 2R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(x, y, z) = (2, -2, 3)$