Exam Review 1-14

1. $$\lim_{x\to -8} \frac{x^2+5x-24}{5x+40} = \lim_{x\to -8} \frac{\cancel{(x+8)}(x-3)}{5\cancel{(x+8)}} = \frac{-8-3}{5} = \frac{-11}{5}$$

2. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2x+2h+1} + \sqrt{2x+1}}{\sqrt{2x+2h+1} + \sqrt{2x+1}}$$

$$= \lim_{h\to 0} \frac{(2x+2h+1)-(2x+1)}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \frac{2}{\sqrt{2x+2\cdot 0+1} + \sqrt{2x+1}}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{2x+1}}$$

$$= \frac{1}{\sqrt{2x+1}}$$

3. $$y = \underbrace{(2x+1)^{2/3}}_{u}\underbrace{(x^3-3x^2)}_{v}$$

$$y' = uv' + vu'$$

$$= (2x+1)^{2/3}(3x^2-6x) + (x^3-3x^2)\tfrac{2}{3}(2x+1)^{-1/3}(2)$$

$$y'\Big|_{x=2} = (2\cdot 2+1)^{2/3}(3\cdot 2^2 - 6\cdot 2) + \frac{4(2^3-3\cdot 2^2)}{3(2\cdot 2+1)^{1/3}}$$

$$= 5^{2/3}\cdot 0 + \frac{4(-4)}{3\cdot 5^{1/3}} = \frac{-16}{3\cdot 5^{1/3}} \text{ or } \frac{-16}{3\sqrt[3]{5}}$$

4. $y = \dfrac{8x^2 + 3}{5x+1}$ $\begin{matrix} u \\ v \end{matrix}$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{(5x+1)(16x) - (8x^2+3)(5)}{(5x+1)^2}$$

$$= \frac{80x^2 + 16x - 40x^2 - 15}{(5x+1)^2}$$

$$= \frac{40x^2 + 16x - 15}{(5x+1)^2}$$

5. $\cos(xy) - \sin(3y) = 1 + x^3$

$$-\sin(xy)\left[x\frac{dy}{dx} + y(1)\right] - \cos(3y) \cdot 3\frac{dy}{dx} = 3x^2$$

$$-x\sin(xy)\frac{dy}{dx} - y\sin(xy) - 3\cos(3y)\frac{dy}{dx} = 3x^2$$

$$-x\sin(xy)\frac{dy}{dx} - 3\cos(3y)\frac{dy}{dx} = 3x^2 + y\sin(xy)$$

$$\left[-x\sin(xy) - 3\cos(3y)\right]\frac{dy}{dx} = 3x^2 + y\sin(xy)$$

$$\frac{dy}{dx} = \frac{3x^2 + y\sin(xy)}{-x\sin(xy) - 3\cos(3y)}$$

6. $y = \ln[x^3(x^2+4)] = \ln(x^3) + \ln(x^2+4) = 3\ln x + \ln(x^2+4)$

note: it's not necessary to use log rules to simplify y but it does make it simpler to find y'

$m = y'\big|_{x=1}$

$y' = 3 \cdot \frac{1}{x} + \frac{1}{x^2+4}(2x) = \frac{3}{x} + \frac{2x}{x^2+4}$

$y'\big|_{x=1} = \frac{3}{1} + \frac{2(1)}{1^2+4} = \frac{17}{5}$

at $x=1$, $y = \ln[1^3(1^2+4)] = \ln 5$

$y = mx + b$, solve for b using $m = \frac{17}{5}$ and the point $(1, \ln 5)$

$\ln 5 = \frac{17}{5}(1) + b$

$b = \ln 5 - \frac{17}{5}$

so $y = \frac{17}{5}x + \ln 5 - \frac{17}{5}$

7. $e^x = \cos x + 1$

$\underbrace{e^x - \cos x - 1}_{f(x)} = 0$

$f'(x) = e^x + \sin x$

RADIANS!

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ so $x_2 = -3 - \frac{e^{-3} - \cos(-3) - 1}{e^{-3} + \sin(-3)} \approx 2.56$

8. $x = e^{-t^2+8t}$

$$v_x = e^{-t^2+8t}(-2t+8)$$

$$v_x\Big|_{t=0.2} = e^{-0.2^2+8(0.2)}(-2\cdot 0.2+8)$$

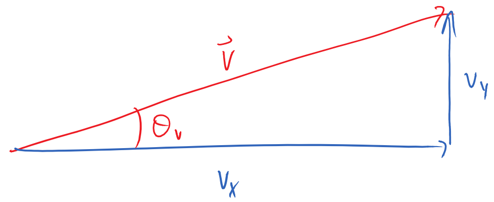
$$= 7.6\,e^{1.56} \approx 36$$

$y = te^{7t}$

$$v_y = te^{7t}\,7 + e^{7t}(1)$$

$$v_y\Big|_{t=0.2} = 7(0.2)e^{7(0.2)} + e^{7(0.2)}$$

$$= 2.4e^{1.4} \approx 10$$



$$v = \sqrt{v_x^2 + v_y^2}$$

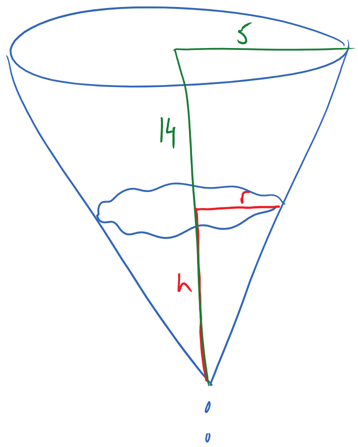
$$= \sqrt{(7.6e^{1.56})^2 + (2.4e^{1.4})^2}$$

$$\approx 37.5$$

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{2.4e^{1.4}}{7.6e^{1.56}}\right) \approx 15.1^\circ$$

$$\vec{v} = 37.5 \text{ m/s @ } 15.1^\circ$$

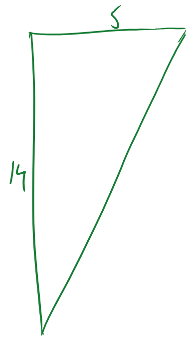
9.



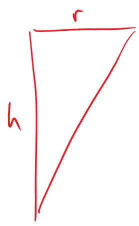
given: $\dfrac{dV}{dt} = 2 \dfrac{m^3}{h}$

wanted: $\left.\dfrac{dh}{dt}\right|_{h=6}$

$V = \frac{1}{3}\pi r^2 h \longleftarrow$ 2 variables!



and



are similar triangles so $\dfrac{r}{5} = \dfrac{h}{14}$

i.e. $r = \dfrac{5h}{14}$

therefore $V = \frac{1}{3}\pi\left(\frac{5h}{14}\right)^2 h$

$$V = \frac{25}{588}\pi h^3$$

$$\frac{dV}{dt} = \frac{25}{196}\pi h^2 \frac{dh}{dt}$$

at $h=6$:

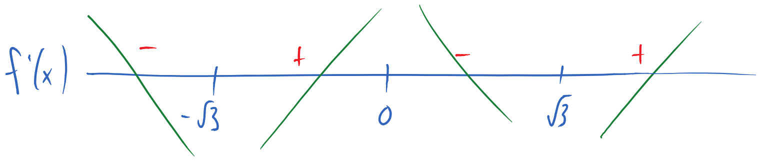
$$2 = \frac{25}{196}\pi (6)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2 \cdot 196}{25(36)\pi} = \frac{98}{225\pi} \approx 0.14 \frac{m}{h}$$

10. $f(x) = x^8 - 4x^6$

a) $f'(x) = 8x^7 - 24x^5 = 8x^5(x^2 - 3) = 8x^5(x+\sqrt{3})(x-\sqrt{3})$

$f'(x) = 0$ when $x = 0, \pm\sqrt{3}$



$f'(-2) = \underset{-}{8(-2)^5}\underset{+}{\left((-2)^2 - 3\right)}$

rel. min at $x = \pm\sqrt{3}$

$f(\sqrt{3}) = (\sqrt{3})^8 - 4(\sqrt{3})^6 = -27$

$f(-\sqrt{3}) = -27$

$\Big\}\ (\pm\sqrt{3}, -27)$

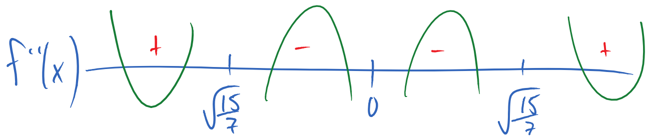
rel. max at $x = 0$

$f(0) = 0$

$\Big\}\ (0,0)$

b) $f''(x) = 56x^6 - 120x^4 = 56x^4\left(x^2 - \frac{15}{7}\right) = 56x^4\left(x + \sqrt{\frac{15}{7}}\right)\left(x - \sqrt{\frac{15}{7}}\right)$

$f''(x) = 0$ when $x = 0, \pm\sqrt{\frac{15}{7}}$



$$f''(-2) = \underset{+}{56(-2)^4}\left(\underset{+}{(-2)^2 - \frac{15}{7}}\right)$$

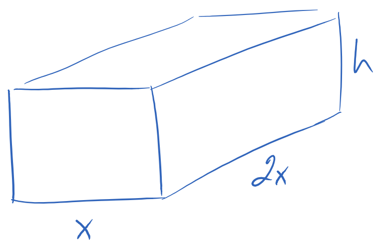
points of inflection at $x = \pm\sqrt{\frac{15}{7}}$

$f\left(\sqrt{\frac{15}{7}}\right) = -\frac{43875}{2401} \approx -18.27$

$f\left(-\sqrt{\frac{15}{7}}\right) \approx -18.27$

$\left(\pm\sqrt{\frac{15}{7}}, -18.27\right)$

11.



we want to maximize volume

$$V = x(2x)(h)$$
$$V = 2x^2h \leftarrow 2 \text{ variables!}$$

$$x + 2x + h = 140$$
$$h = 140 - 3x$$

$$V = 2x^2(140 - 3x)$$

$$V = 280x^2 - 6x^3$$

$$V' = 560x - 18x^2$$
$$= 2x(280 - 9x)$$

$V' = 0$ when $\cancel{x = 0}$ or $x = \frac{280}{9}$ ← this is the only reasonable solution so it must give the max

when $x = \frac{280}{9}$,

$$V = 280\left(\frac{280}{9}\right)^2 - 6\left(\frac{280}{9}\right)^3 \approx 90337.45 \text{ cm}^3$$

12. $f(x) \approx f(a) + f'(a)(x-a)$

$\sin\left(\frac{5\pi}{18}\right)$

to choose a : we need the closest special angle to $\frac{5\pi}{18} = 50^\circ$

let $a = \frac{\pi}{4} \leftarrow 45^\circ$

$f(x) = \sin x \qquad f(a) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$f'(x) = \cos x \qquad f'(a) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

so $f(x) \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$ for $x \approx \frac{\pi}{4}$

$\sin\frac{5\pi}{18} \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(\frac{5\pi}{18} - \frac{\pi}{4}\right) \approx 0.77$

13. $f(x) = \csc^2(2x) + \tan^{-1}(5x)$

$f'(x) = 2\csc(2x)\left[-\csc(2x)\cot(2x)\right](2) + \frac{1}{1+(5x)^2}(5)$

$f'\left(\frac{\pi}{6}\right) = -4\csc^2\left(2\cdot\frac{\pi}{6}\right)\cot\left(2\cdot\frac{\pi}{6}\right) + \frac{5}{1+\left(5\cdot\frac{\pi}{6}\right)^2} \approx -2.4$

14. $f(x) = \log_2\left(x^2 + 5x + 1\right) + 2^{4x}$

$f'(x) = \frac{1}{(x^2+5x+1)\ln 2}(2x+5) + 2^{4x}(4)\ln 2$

$f'(0) = \frac{1}{(0^2+5\cdot 0+1)\ln 2}(2\cdot 0+5) + 2^{4\cdot 0}(4)\ln 2$

$= \frac{5}{\ln 2} + 4\ln 2$