

## 27.1 Derivatives of Sine and Cosine

$$\frac{d}{dx} [\sin x] = \cos x$$

$$\frac{d}{dx} [\cos x] = -\sin x$$

Recall Chain Rule

$$\frac{d}{dx} (1+4x)^2 = 2(1+4x) \cdot 4$$

$$\begin{aligned}\frac{d}{dx} \sin x^2 &= \cos x^2 \cdot 2x \\ &= 2x \cos x^2\end{aligned}$$

$$\frac{d}{dx} \sin^2 x = \frac{d}{dx} [\sin x]^2$$

$$= 2 \sin x \cdot \cos x$$

Ex: Find  $f'(x)$

a)  $f(x) = 4 \sin(7x + 2)$

$$\begin{aligned}f'(x) &= 4 \cos(7x + 2) \cdot 7 \\&= 28 \cos(7x + 6)\end{aligned}$$

b)  $f(x) = \cos^4 x$

$$f'(x) = [\cos x]^4$$

$$\begin{aligned}f'(x) &= 4(\cos x)^3 (-\sin x) \\&= -4 \cos^3 x \sin x\end{aligned}$$

c)  $f(x) = \cos^2 \left(1 - \frac{\pi x}{2}\right)$

$$f'(x) = [\cos \left(1 - \frac{\pi x}{2}\right)]^2$$

$$\begin{aligned}f'(x) &= 2 \cos \left(1 - \frac{\pi x}{2}\right) \frac{d}{dx} \cos \left(1 - \frac{\pi x}{2}\right) \\&= 2 \cos \left(1 - \frac{\pi x}{2}\right) [-\sin \left(1 - \frac{\pi x}{2}\right)] \left(-\frac{\pi}{2}\right)\end{aligned}$$

$$= \pi \cos \left(1 - \frac{\pi x}{2}\right) \sin \left(1 - \frac{\pi x}{2}\right)$$

$$d) f(x) = x \cos 7x$$

$$\begin{aligned}f'(x) &= x(-\sin 7x \cdot 7) + \cos 7x \cdot 1 \\&= -7x \sin 7x + \cos 7x\end{aligned}$$

$$e) f(x) = \frac{\sin x}{1+x^2}$$

$$\begin{aligned}f'(x) &= \frac{(1+x^2)\cos x - \sin x(2x)}{(1+x^2)^2} \\&= \frac{\cos x + x^2 \cos x - 2x \sin x}{(1+x^2)^2}\end{aligned}$$

$$\text{Ex: Find } \frac{dy}{dx} \quad \sin(xy) + \cos 3y = x^3$$

$$\cos(xy) \frac{d}{dx}(xy) - 3\sin 3y \frac{dy}{dx} = 3x^2$$

$$\cos(xy) \left[ x \frac{dy}{dx} + y \right] - 3\sin 3y \frac{dy}{dx} = 3x^2$$

$$x \cos(xy) \frac{dy}{dx} + y \cos(xy) - 3\sin 3y \frac{dy}{dx} = 3x^2$$

$$\left[ x \cos(xy) - 3\sin(3y) \right] \frac{dy}{dx} = 3x^2 - y \cos(xy)$$

$$\frac{dy}{dx} = \frac{3x^2 - y \cos(xy)}{x \cos(xy) - 3\sin(3y)}$$