1. [5 marks] State whether each series below converges or diverges. If it converges, find the sum. Give exact values.

a) $3 - \frac{3}{5} + \frac{3}{25} - \frac{3}{125} + \ldots$

Geometric $a=3$ $r=-\frac{1}{5}$

$\boxed{\text{Converges}}$ $S = \frac{a}{1-r} = \frac{3}{1-(-\frac{1}{5})} = \frac{3}{\left(\frac{6}{5}\right)} = \frac{15}{6}$ or $\frac{5}{2}$

if $-1<r<1$

b) $7 + \frac{7}{3} + \frac{7}{9} + \frac{7}{27} + \ldots$

Geometric $a=7$ $r=\frac{1}{3}$

$\boxed{\text{Converges}}$ $S = \frac{a}{1-r} = \frac{7}{1-\frac{1}{3}} = \frac{7}{\left(\frac{2}{3}\right)} = \frac{21}{2}$

c) $1+2+1+2+1+2+\ldots$

Not geometric, so look at partial sums.

Partial sums:

$S_1 = 1$

$S_2 = 1+2 = 3$

$S_3 = 1+2+1 = 4$

$S_4 = 1+2+1+2 = 6$

$S_n \to \infty$ as $n \to \infty$

$\boxed{\text{Diverges}}$

2. [6 marks] Use your formula sheet to find the first three nonzero terms of the Maclaurin series for each function below.

a) $x\sin(\frac{x}{3})$

$$\sin y = y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$$

$$\sin\left(\frac{x}{3}\right) = \frac{x}{3} - \frac{\left(\frac{x}{3}\right)^3}{3!} + \frac{\left(\frac{x}{3}\right)^5}{5!} - \dots$$

$$\sin\left(\frac{x}{3}\right) = \frac{x}{3} - \frac{x^3}{162} + \frac{x^5}{29160} - \dots$$

$$x\sin\left(\frac{x}{3}\right) = \frac{x^2}{3} - \frac{x^4}{162} + \frac{x^6}{29160} - \dots$$

(-1) if you did $x \sin x$ and then subbed $\frac{x}{3}$ for x.

b) $(1+2x^2)e^{-3x}$

$$e^y = 1 + y + \frac{y^2}{2!} + \dots$$

$$e^{-3x} = 1 + (-3x) + \frac{(-3x)^2}{2!} + \dots$$

$$e^{-3x} = 1 - 3x + \frac{9}{2}x^2 + \dots$$

$$(1+2x^2)e^{-3x} = (1+2x^2)\left(1 - 3x + \frac{9}{2}x^2 + \dots\right)$$

$$= 1 - 3x + \frac{9}{2}x^2 + \dots$$

$$+ 2x^2 + \dots$$

$$= 1 - 3x + \frac{13}{2}x^2 + \dots$$

3. [2 marks] For what values of y does the geometric series below converge?

$1 + \frac{y}{4} + \frac{y^2}{16} + \frac{y^3}{64} + \ldots$

ratio $r = \frac{y}{4}$

Series converges when $-1 < r < 1$

$\boxed{-1 < \frac{y}{4} < 1}$

or $\boxed{-4 < y < 4}$

4. [5 marks] Use your formula sheet to find the first three nonzero terms of the Maclaurin series for $f(x) = (1+x^3)^5$.

$$(1+y)^5 = 1 + 5y + \frac{5(4)}{2!}y^2 + \cdots$$

$$(1+y)^5 = 1 + 5y + 10y^2 + \cdots$$

$$(1+x^3)^5 = 1 + 5x^3 + 10(x^3)^2 + \cdots$$

$$(1+x^3)^5 = 1 + 5x^3 + 10x^6 + \cdots$$

b) Use part a) to approximate $\int_0^{0.3} (1+x^3)^5 \, dx$. Round your answer to four decimal places.

$$\int_0^{0.3} (1+x^3)^5 \, dx \approx \int_0^{0.3} (1 + 5x^3 + 10x^6) \, dx$$

$$= \left[x + \frac{5}{4}x^4 + \frac{10}{7}x^7 \right]_0^{0.3}$$

$$= 0.3 + \frac{5}{4}(0.3)^4 + \frac{10}{7}(0.3)^7 - (0)$$

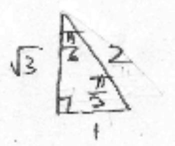
$$\approx 0.3104$$

5. [7 marks] a) Find the first three nonzero terms of the Taylor series for $f(x) = \sec x$ centred at $x = \frac{\pi}{3}$.

k	$f^{(k)}(x)$	$f^{(k)}(\frac{\pi}{3})$	$\frac{f^{(k)}(\frac{\pi}{3})}{k!}$
0	$\sec x$	$\sec\frac{\pi}{3} = 2$	2
1	$\sec x \tan x$	$2\sqrt{3}$	$2\sqrt{3}$
2	$\sec x(\sec^2 x) + \tan x(\sec x \tan x)$ $= \sec^3 x + \tan^2 x \cdot \sec x$	$2^3 + (\sqrt{3})^2 \cdot 2$ $= 14$	7

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\boxed{\sec\frac{\pi}{3} = 2}$ $\quad$ $\boxed{\tan\frac{\pi}{3} = \sqrt{3}}$



Answer:

$$\boxed{\sec x = 2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 7\left(x - \frac{\pi}{3}\right)^2 + \ldots}$$

b) Use part a) to approximate $\sec(\frac{2\pi}{7})$. Round your answer to four decimal places.

$$\sec\left(\frac{2\pi}{7}\right) = 2 + 2\sqrt{3}\left(\frac{2\pi}{7} - \frac{\pi}{3}\right) + 7\left(\frac{2\pi}{7} - \frac{\pi}{3}\right)^2 + \ldots$$

$$\approx 1.6384$$