1. [5 marks] Evaluate $\int \frac{7+x}{3x^2-4x}\,dx$

$$\frac{7+x}{x(3x-4)} = \frac{A}{x} + \frac{B}{3x-4}$$

$$7+x = A(3x-4) + Bx$$

Sub $x=0$: $\quad 7 = -4A \qquad \boxed{A = -7/4}$

Sub $x=\frac{4}{3}$: $\quad 7+\frac{4}{3} = \frac{4}{3}B$

$$\frac{25}{3} = \frac{4}{3}B$$

$$\boxed{\frac{25}{4} = B}$$

$$\int \frac{7+x}{3x^2-4x}\,dx$$

$$= \int\left(-\frac{7}{4}\cdot\frac{1}{x} + \frac{25}{4}\frac{1}{3x-4}\right)dx$$

$$= \int\left(-\frac{7}{4}\frac{1}{x} + \frac{25}{12}\frac{3}{3x-4}\right)dx$$

$$= -\frac{7}{4}\ln|x| + \frac{25}{12}\ln|3x-4| + C$$

2. [6 marks] Evaluate $\int \frac{9x+2}{(x^2-1)(x-2)}\,dx$

(-2) if you had only A

$$\frac{9x+2}{(x^2+1)(x-2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-2}$$

$$9x+2 = (Ax+B)(x-2) + C(x^2+1)$$

Sub $x=2$: $\quad 20 = 5C \quad \boxed{C=4}$

Expand out to compare coefficients:

$$9x+2 = Ax^2 - 2Ax + Bx - 2B + Cx^2 + C$$

$[x^2]$ $\quad 0 = A + C$

$C = 4 \quad \boxed{A=-4}$

Constants:

$$2 = -2B + C$$

$C=4 \quad 2 = -2B+4$

$\boxed{B=1}$

$$\text{Integral} = \int \left(\frac{-4x+1}{x^2+1} + \frac{4}{x-2}\right)dx$$

$$= \int \left(\frac{-4x}{x^2+1} + \frac{1}{x^2+1} + \frac{4}{x-2}\right)dx$$

$-2 \cdot \frac{2x}{x^2+1}$

$$= -2\ln|x^2+1| + \tan^{-1}x + 4\ln|x-2| + C$$

3. [6 marks] Evaluate $\int \frac{dx}{x\sqrt{4-x^2}}$

$$= \int \frac{2\cos\theta \, d\theta}{2\sin\theta \cdot 2\cos\theta}$$

Sub $x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

$$= \int \frac{1}{2} \frac{1}{\sin\theta} d\theta$$

$$= \int \frac{1}{2} \csc\theta \, d\theta$$

$$= \frac{1}{2} \ln|\csc\theta - \cot\theta| + C$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{x}{2}\right)} = \frac{2}{x}$$

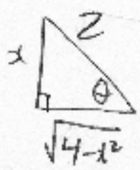
$$= \frac{1}{2} \ln\left| \frac{2}{x} - \frac{\sqrt{4-x^2}}{x} \right| + C$$

$$\cot\theta = \frac{2\cos\theta}{2\sin\theta} = \frac{\sqrt{4-x^2}}{x}$$

(-1) if you wrote

$\csc\left(\sin^{-1}\left(\frac{x}{2}\right)\right)$

or draw



4. [3 marks] Convert the following equation to cylindrical coordinates: $z = 4x^2 - 7x + 4y^2$. Simplify your answer.

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$z = 4(r\cos\theta)^2 - 7r\cos\theta + 4(r\sin\theta)^2$$

$$z = 4r^2\cos^2\theta - 7r\cos\theta + 4r^2\sin^2\theta$$

$$z = 4r^2(\cos^2\theta + \sin^2\theta) - 7r\cos\theta$$

$$z = 4r^2 - 7r\cos\theta$$

or $\quad z = r(4r - 7\cos\theta)$

5. [4 marks] Let $z = 4x^3y^2 - xy^3$. Find the following partial derivatives:

a) $\frac{\partial z}{\partial x} = 12x^2y^2 - y^3$

b) $\frac{\partial z}{\partial y} = 8x^3y - 3xy^2$

c) $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$

$$= \frac{\partial}{\partial y}(12x^2y^2 - y^3)$$

$$= 24x^2y - 3y^2$$

d) $\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$

$$= \frac{\partial}{\partial y}(8x^3y - 3xy^2)$$

$$= 8x^3 - 6xy$$

6. [6 marks] Evaluate $\int_1^2 \int_0^x (xy + y^2 + 2)\, dy\, dx$

$$= \int_1^2 \left[\frac{xy^2}{2} + \frac{y^3}{3} + 2y\right]_0^x dx$$

$$= \int_1^2 \left[\left(\frac{x \cdot x^2}{2} + \frac{x^3}{3} + 2x\right) - (0)\right] dx$$

$$= \int_1^2 \left(\underbrace{\frac{x^3}{2} + \frac{x^3}{3}} + 2x\right) dx$$

$$= \int_1^2 \left(\frac{5}{6}x^3 + 2x\right) dx$$

$$= \left[\frac{5}{24}x^4 + x^2\right]_1^2$$

$$= \left[\frac{5}{24}(2)^4 + (2)^2\right] - \left[\frac{5}{24} + 1\right]$$

$$= 6.125$$

or $\frac{49}{8}$