1. [4 marks] Evaluate $\int_0^{\pi} \frac{\sin t}{2+\cos t}\,dt$. Give an exact value.

Sub $u = 2+\cos t$

$du = -\sin t\,dt$

$-du = \sin t\,dt$

When $t=0$ $u=3$

$t=\pi$ $u=1$

$$= -\int_3^1 \frac{du}{u}$$

$$= -\left[\ln|u|\right]_3^1$$

$$= -\ln 1 + \ln 3$$

$$= \ln 3 \qquad [\ln 1 = 0]$$

$\cos 0 = 1$

$\cos \pi = -1$

2. [4 marks] Evaluate $\int_0^1 (12 - 16x^3)\, e^{6x - 2x^4}\,dx$. Give an exact value.

Sub $u = 6x - 2x^4$

$du = (6 - 8x^3)\,dx$

$2\,du = (12 - 16x^3)\,dx$

When $x=0$ $u=0$

$x=1$ $u=4$

$$= 2\int_0^4 e^u\,du$$

$$= \left[2e^u\right]_0^4$$

$$= 2e^4 - 2e^0$$

$$= 2e^4 - 2$$

3. [3 marks] Evaluate $\int \frac{2}{\sqrt{81-16x^2}}\,dx$

$$= \int \frac{-2}{\sqrt{9^2-(4x)^2}}\,dx$$

Sub $u = 4x$

$du = 4\,dx$

$\frac{1}{4}du = dx$

$$= \frac{1}{4}\int \frac{-2}{\sqrt{9^2-u^2}}\,du$$

$$= \frac{1}{4}\left(-2\sin^{-1}\left(\frac{u}{9}\right)\right) + C$$

$$= -\frac{1}{2}\sin^{-1}\left(\frac{u}{9}\right) + C$$

$$= -\frac{1}{2}\sin^{-1}\left(\frac{4x}{9}\right) + C$$

4. [4 marks] Evaluate $\int \frac{(12+e^{-3x})^4}{e^{3x}}\,dx$

$$= \int (12+e^{-3x})^4 e^{-3x}\,dx$$

Sub $u = 12+e^{-3x}$

$du = -3e^{-3x}\,dx$

$-\frac{1}{3}du = e^{-3x}\,dx$

$$= -\frac{1}{3}\int u^4\,du$$

$$= -\frac{1}{15}u^5 + C$$

$$= -\frac{1}{15}(12+e^{-3x})^5 + C$$

5. [4 marks] Evaluate $\int \frac{x}{\sqrt{100-x^2}}\,dx$

Sub $u = 100 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\int \frac{du}{\sqrt{u}}$

$= -u^{1/2} + C$

$= -\sqrt{100-x^2} + C$

6. [4 marks] Evaluate $\int -8x\sin x\,dx$

Integration by Parts

	D	I
⊕	$-8x$	$\sin x$
⊖	-8	$-\cos x$
	0	$-\sin x$

$\int -8x\sin x\,dx$

$= 8x\cos x - 8\sin x + C$

or use $\int u\,dv = uv - \int v\,du$

with $u = -8x$ $dv = \sin x\,dx$

same answer

7. [3 marks] Evaluate $\int \frac{7}{5+9x^2}\,dx$

$$= \int \frac{7\,dx}{\sqrt{5}^2+(3x)^2}$$

Sub $u = 3x$
$du = 3dx$
$\frac{1}{3}du = dx$

$$= \frac{7}{3}\int \frac{du}{\sqrt{5}^2+u^2}$$

$$= \frac{7}{3}\left(\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{u}{\sqrt{5}}\right)\right)+C$$

$$= \frac{7}{3\sqrt{5}}\tan^{-1}\left(\frac{3x}{\sqrt{5}}\right)+C$$

or $\frac{7\sqrt{5}}{15}\tan^{-1}\left(\frac{3\sqrt{5}x}{5}\right)+C$

8. [4 marks] Evaluate $\int \sin^5 x\,dx$

$$= \int \sin^4 x \cdot \sin x\,dx$$
$$= \int (\sin^2 x)^2 \cdot \sin x\,dx$$
$$= \int (1-\cos^2 x)^2 \sin x\,dx$$
$$= \int (1 - 2\cos^2 x + \cos^4 x)\sin x\,dx$$

Sub $u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$$= -\int (1 - 2u^2 + u^4)\,du$$
$$= -\left(u - \frac{2}{3}u^3 + \frac{u^5}{5}\right)+C$$
$$= -\left(\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x\right)+C$$