Math 187 Test One Total: 30 marks

Name: ______________________________

Show all your work for full marks.

1. [6 marks] A flare is ejected vertically upwards from the ground at an initial velocity of 15 m/s.

a) Find a formula for the flare's height $h(t)$.

$a(t) = -9.8 \text{ m/s}^2$

$v(t) = \int -9.8\, dt$

$v(t) = -9.8t + C_1$

$v(0) = 15$: $15 = C_1$

$v(t) = -9.8t + 15$

$h(t) = \int (-9.8t + 15)\, dt$

$h(t) = -9.8\frac{t^2}{2} + 15t + C_2$

$h(0) = 0$: $0 = C_2$

$\boxed{h(t) = -4.9t^2 + 15t}$

b) Find the flare's maximum height. Round your answer to one decimal place.

Maximum height occurs when $v = 0$

Set $v = 0$: $0 = -9.8t + 15$

$9.8t = 15$

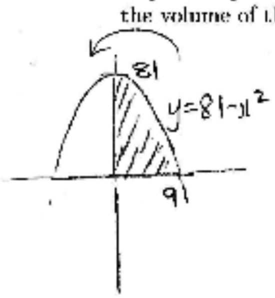
$t = \frac{15}{9.8}$

$h(t) = -4.9t^2 + 15t$

Maximum height is $h\left(\frac{15}{9.8}\right) = -4.9\left(\frac{15}{9.8}\right)^2 + 15\left(\frac{15}{9.8}\right)$

$\approx 11.5 \text{ m}$

2. [5 marks] Consider the first-quadrant region bounded by $y = 81 - x^2$. Find the volume of the solid generated by revolving the region about the y-axis.

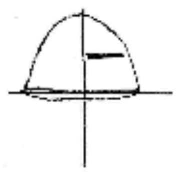


Disk Method

$dV = \pi \cdot \text{radius}^2 \cdot \text{thickness}$

$dV = \pi x^2 \, dy$

$V = \pi \int_0^{81} x^2 \, dy$



$y = 81 - x^2 \nearrow$

$x^2 = 81 - y$

$V = \pi \int_0^{81} (81 - y) \, dy$

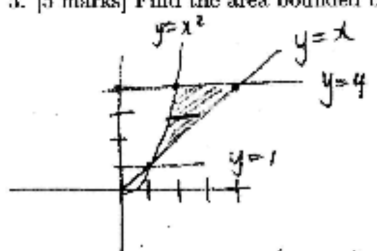
$= \pi \left[81y - \frac{y^2}{2} \right]_0^{81}$

$= \pi \left[\left(81^2 - \frac{81^2}{2} \right) - 0 \right]$

$= \underline{\frac{6561\pi}{2}}$

Shell Method gives same answer.

3. [5 marks] Find the area bounded by $y = x$, $y = x^2$, $y = 1$ and $y = 4$.



$A = \int_1^4 (x_r - x_l) \, dy$

$x_r: \; y = x$

$x_r = y \nearrow$

$x_l: \; y = x^2$

$x = \pm\sqrt{y}$

$x_l = \sqrt{y} \nearrow$

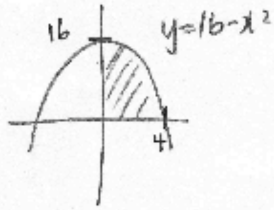
$= \int_1^4 (y - \sqrt{y}) \, dy$

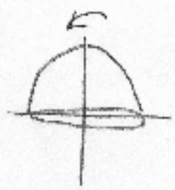
$= \left[\frac{y^2}{2} - \frac{2}{3} y^{3/2} \right]_1^4$

$= \left[\frac{16}{2} - \frac{2}{3} \cdot 8 \right] - \left[\frac{1}{2} - \frac{2}{3} \right]$

$= \frac{17}{6}$

4. [6 marks] Consider the first-quadrant region bounded by $y = 16 - x^2$ and the axes. A solid is produced by revolving this region about the y-axis. The volume of the solid is 128π. Find the centroid of the solid.





$\boxed{\bar{x}=0}$ by symmetry

$$\bar{y} = \frac{\int_0^{16} y x^2 \, dy}{\int_0^{16} x^2 \, dy}$$

$$\pi \int_0^{16} x^2 \, dy = \text{Volume}$$

Given $\pi \int_0^{16} x^2 \, dy = 128\pi$

$$\int_0^{16} x^2 \, dy = 128$$

[2] or $B = \int_0^{16} x^2 \, dy = \int_0^{16} (16-y) \, dy$

$y = 16 - x^2$ $\quad = \left(16y - \frac{y^2}{2}\right]_0^{16}$

$x^2 = 16 - y$ $\quad = 128$

$$\bar{y} = \frac{1}{B} \int_0^{16} y x^2 \, dy$$

$$= \frac{1}{128} \int_0^{16} y x^2 \, dy$$

$$= \frac{1}{128} \int_0^{16} y(16-y) \, dy$$

$$= \frac{1}{128} \int_0^{16} (16y - y^2) \, dy$$

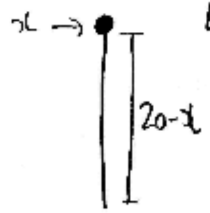
$$= \frac{1}{128} \left[8y^2 - \frac{y^3}{3} \right]_0^{16}$$

$$= \frac{1}{128} \left[\frac{2048}{3} \right]$$

$$= \frac{16}{3}$$

Centroid $(\bar{x}, \bar{y}) = \left(0, \frac{16}{3}\right)$

(5.) [4 marks] Find the work done by winding up 18m of a 20m-long rope on which the force of gravity is 8 N/m.



Let x = length of rope already wound up

$0 \le x \le 18$

$N/m \cdot m = N$ ✓

$$F(x) = 8(20-x)$$

$$W = \int_0^{18} F(x)\,dx$$

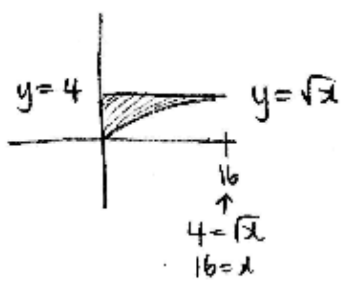
$$= \int_0^{18} 8(20-x)\,dx$$

$$= \int_0^{18} (160-8x)\,dx$$

$$= \left[160x - 4x^2\right]_0^{18}$$

$$= 1584\ J$$

6. [4 marks] A thin, flat metal plate has density k. The plate is bounded by $y = \sqrt{x}$, $x = 0$ and $y = 4$. Find the moment of inertia of the plate with respect to the y-axis.



$$I_y = k\int_0^{16} x^2 (y_t - y_b)\,dx$$

$$= k\int_0^{16} x^2 (4-\sqrt{x})\,dx$$

$$= k\int_0^{16} (4x^2 - x^{5/2})\,dx$$

$$= k\left[\frac{4}{3}x^3 - \frac{2}{7}x^{7/2}\right]_0^{16}$$

$$= \frac{16384k}{21}$$