

$$\textcircled{1} \text{ a) } AB = \begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -14 & -18 \\ -22 & -26 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 13 & -42 \end{bmatrix}$$

$$\text{b) } AB = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix} \begin{bmatrix} -2 & 5 \\ 1 & 6 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 8 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 5 \\ 1 & 6 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 18 & -13 & 11 \\ 25 & -19 & 20 \\ 13 & -11 & 16 \end{bmatrix}$$

$$\text{c) } AB = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 13 \\ -8 \end{bmatrix} = \begin{bmatrix} -20 \\ -27 \end{bmatrix}$$

BA is undefined

$\begin{matrix} B & A \\ 3 \times 1 & 2 \times 3 \\ \uparrow & \uparrow \\ \text{must be equal} \end{matrix}$

② If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
(provided $ad-bc \neq 0$)

a) $ad-bc = 1(-7) - (-6)(4) = 17$

$$A^{-1} = \frac{1}{17} \begin{bmatrix} -7 & 6 \\ -4 & 1 \end{bmatrix}$$

b) $ad-bc = 12(2) - (-3)(8) = 0$

B^{-1} does not exist.

$$\textcircled{3} \begin{bmatrix} 2 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & 3 & 1 & | & 0 & 1 & 0 \\ 2 & 2 & 2 & | & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 \begin{bmatrix} \textcircled{1} & 3 & 1 & | & 0 & 1 & 0 \\ 2 & 1 & 1 & | & 1 & 0 & 0 \\ 2 & 2 & 2 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array} \begin{bmatrix} 1 & 3 & 1 & | & 0 & 1 & 0 \\ 0 & -5 & -1 & | & 1 & -2 & 0 \\ 0 & -4 & 0 & | & 0 & -2 & 1 \end{bmatrix}$$

$$R_2 / (-5) \begin{bmatrix} 1 & 3 & 1 & | & 0 & 1 & 0 \\ 0 & \textcircled{1} & 1/5 & | & -1/5 & 2/5 & 0 \\ 0 & -4 & 0 & | & 0 & -2 & 1 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 3R_2 \\ R_3 + 4R_2 \end{array} \begin{bmatrix} 1 & 0 & 2/5 & | & 3/5 & -1/5 & 0 \\ 0 & 1 & 1/5 & | & -1/5 & 2/5 & 0 \\ 0 & 0 & 4/5 & | & -4/5 & -2/5 & 1 \end{bmatrix}$$

$$R_3 \times \frac{5}{4} \begin{bmatrix} 1 & 0 & 2/5 & | & 3/5 & -1/5 & 0 \\ 0 & 1 & 1/5 & | & -1/5 & 2/5 & 0 \\ 0 & 0 & \textcircled{1} & | & -1 & -1/2 & 5/4 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 2/5 R_3 \\ R_2 - 1/5 R_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & -1/2 \\ 0 & 1 & 0 & | & 0 & 1/2 & -1/4 \\ 0 & 0 & 1 & | & -1 & -1/2 & 5/4 \end{bmatrix}$$

$$\leftarrow A^{-1} = \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1/2 & -1/4 \\ -1 & -1/2 & 5/4 \end{bmatrix}$$

$$\textcircled{4} \text{ a) } \begin{bmatrix} 8 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \end{bmatrix} \quad (AX=B)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 17 \\ -5 \end{bmatrix} \quad (X=A^{-1}B)$$

$$\begin{bmatrix} 8 & -1 \\ -4 & 1 \end{bmatrix}^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} 17 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 12 \\ 28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

b) To solve $AX=B$, use $X=A^{-1}B$

To find A^{-1} :

$$\left[\begin{array}{ccc|ccc} 4 & 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|ccc} \textcircled{1} & 1 & 2 & 0 & 1 & 0 \\ 4 & 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_2 - 4R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -4 & -4 & 1 & -4 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array} \right]$$

$$R_2 / (-4) \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & \textcircled{1} & 1 & -1/4 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array} \right]$$

$$R_1 - R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 1 & 1 & -1/4 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array} \right]$$

$$R_3 / (-1) \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 1 & 1 & -1/4 & 1 & 0 \\ 0 & 0 & \textcircled{1} & 0 & 1 & -1 \end{array} \right]$$

$$\begin{array}{l} R_1 - R_3 \\ R_2 - R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & -1 & 1 \\ 0 & 1 & 0 & -1/4 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right]$$

$\uparrow$
 A^{-1}

See next page $\rightarrow$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -1 & 1 \\ -\frac{1}{4} & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix} \quad (x = A^{-1}B)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$(5a) \quad \left[\begin{array}{ccc|c} 1 & 3 & -2 & 9 \\ 2 & -1 & 4 & 6 \\ -3 & 2 & -3 & -1 \end{array} \right]$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 + 3R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 3 & -2 & 9 \\ 0 & -7 & 8 & -12 \\ 0 & 11 & -9 & 26 \end{array} \right]$$

$$R_2 / (-7) \quad \left[\begin{array}{ccc|c} 1 & 3 & -2 & 9 \\ 0 & 1 & -8/7 & 12/7 \\ 0 & 11 & -9 & 26 \end{array} \right]$$

$$\begin{array}{l} R_1 - 3R_2 \\ R_3 - 11R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 10/7 & 27/7 \\ 0 & 1 & -8/7 & 12/7 \\ 0 & 0 & 25/7 & 50/7 \end{array} \right]$$

$$R_3 \times \frac{7}{25} \quad \left[\begin{array}{ccc|c} 1 & 0 & 10/7 & 27/7 \\ 0 & 1 & -8/7 & 12/7 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\begin{array}{l} R_1 - 10R_3 \\ R_2 + 8R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\begin{array}{lcl} x & = & 1 \\ y & = & 4 \\ z & = & 2 \end{array}$$

$$(x, y, z) = (1, 4, 2)$$

$$b) \begin{bmatrix} 3 & -18 & 21 & | & 12 \\ 2 & 7 & -6 & | & 3 \end{bmatrix}$$

$$R_1/3 \begin{bmatrix} ① & -6 & 7 & | & 4 \\ 2 & 7 & -6 & | & 3 \end{bmatrix}$$

$$R_2 - 2R_1 \begin{bmatrix} 1 & -6 & 7 & | & 4 \\ 0 & 19 & -20 & | & -5 \end{bmatrix}$$

$$R_2/19 \begin{bmatrix} 1 & -6 & 7 & | & 4 \\ 0 & ① & -20/19 & | & -5/19 \end{bmatrix}$$

$$R_1 + 6R_2 \begin{bmatrix} 1 & 0 & 13/19 & | & 46/19 \\ 0 & 1 & -20/19 & | & -5/19 \end{bmatrix} \quad \begin{aligned} x + \frac{13}{19}z &= \frac{46}{19} \\ y - \frac{20}{19}z &= \frac{-5}{19} \end{aligned}$$

$$\text{So } x = \frac{46}{19} - \frac{13}{19}z, \quad y = \frac{-5}{19} + \frac{20}{19}z$$

$z = \text{any real number}$

Can also be written

$$(x, y, z) = \left(\frac{46}{19} - \frac{13}{19}t, \frac{-5}{19} + \frac{20}{19}t, t \right)$$

$\uparrow$
any real #

Two particular solutions:

$$(z=0) \quad (x, y, z) = \left(\frac{46}{19}, \frac{-5}{19}, 0 \right)$$

$$(z=1) \quad \left(\frac{33}{19}, \frac{15}{19}, 1 \right)$$

$$c) \begin{bmatrix} \textcircled{1} & 3 & 3 & | & 12 \\ 2 & 20 & 10 & | & 8 \\ 1 & 10 & 5 & | & 0 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - R_1 \end{array} \begin{bmatrix} 1 & 3 & 3 & | & 12 \\ 0 & 14 & 4 & | & -16 \\ 0 & 7 & 2 & | & -12 \end{bmatrix}$$

$$R_2/14 \begin{bmatrix} 1 & 3 & 3 & | & 12 \\ 0 & \textcircled{1} & 2/7 & | & -8/7 \\ 0 & 7 & 2 & | & -12 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 3R_2 \\ R_3 - 7R_2 \end{array} \begin{bmatrix} 1 & 0 & 15/7 & | & 108/7 \\ 0 & 1 & 2/7 & | & -8/7 \\ 0 & 0 & 0 & | & -4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 15/7 & 108/7 \\ 0 & 1 & 2/7 & -8/7 \\ 0 & 0 & 0 & -4 \end{array} \right] \leftarrow \text{impossible}$$

$$0x + 0y + 0z = -4$$

is impossible

No solution

"System is inconsistent."