① $S_1 = \frac{5}{1} = 5$

$S_2 = \frac{5}{1} + \frac{5}{2} = \frac{15}{2}$

$S_3 = \frac{5}{1} + \frac{5}{2} + \frac{5}{3} = \frac{55}{6}$

$S_4 = \frac{5}{1} + \frac{5}{2} + \frac{5}{3} + \frac{5}{4} = \frac{125}{12}$

② a) Geometric $r = \frac{-1}{3}$

$\boxed{\text{Converges}}$ $S = \frac{a}{1-r} = \frac{1}{1-(\frac{-1}{3})} = \frac{1}{(\frac{4}{3})} = \frac{3}{4}$

b) Geometric $r = 5$

$\boxed{\text{Diverges}}$

c) Geometric $r = \frac{1}{4}$

$\boxed{\text{Converges}}$ $S = \frac{a}{1-r} = \frac{4}{1-\frac{1}{4}} = \frac{4}{(\frac{3}{4})} = \frac{16}{3}$

d) Not Geometric

$S_1 = 1$

$S_2 = 1+2 = 3$

$S_3 = 6$

$\vdots$

$S_n \to \infty$ as $n \to \infty$

$\boxed{\text{Diverges}}$

③ a) $(1+y)^{1/2} \approx 1 + \frac{1}{2}y + \frac{\frac{1}{2}(-\frac{1}{2})}{2} y^2$

$(1+y)^{1/2} \simeq 1 + \frac{y}{2} - \frac{y^2}{8}$

$(1+2x)^{1/2} \simeq 1 + \frac{2x}{2} - \frac{(2x)^2}{8}$

$(1+2x)^{1/2} \simeq 1 + x - \frac{x^2}{2}$

b) Want $1+2x = 0.88$

$2x = -0.12$

$\boxed{x = -0.06}$

$(0.88)^{1/2} \simeq 1 + (-0.06) - \frac{(-0.06)^2}{2}$

$= 0.9382$

④ a)

$$e^{y} \approx 1 + y + \frac{y^2}{2}$$

$$e^{x^3} \approx 1 + x^3 + \frac{(x^3)^2}{2}$$

$$e^{x^3} \approx 1 + x^3 + \frac{x^6}{2}$$

So

$$1 - 2x^3 + e^{x^3} \approx 1 - 2x^3 + \left(1 + x^3 + \frac{x^6}{2}\right)$$

$$= 2 - x^3 + \frac{x^6}{2}$$

b)

$$\int_0^{0.2} (1 - 2x^3 + e^{x^2})\,dx \approx \int_0^{0.2} \left(2 - x^3 + \frac{x^6}{2}\right) dx$$

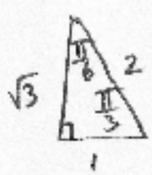
$$= \left[2x - \frac{x^4}{4} + \frac{x^7}{14}\right]_0^{0.2}$$

$$= 2(0.2) - \frac{(0.2)^4}{4} + \frac{(0.2)^7}{14} - (0)$$

$$\approx 0.3996$$

⑤

k	$f^{(k)}(x)$	$f^{(k)}\left(\frac{\pi}{6}\right)$	$\frac{f^{(k)}\left(\frac{\pi}{6}\right)}{k!}$
0	$f(x) = \sin x$	$\frac{1}{2}$	$\frac{1}{2}$
1	$f'(x) = \cos x$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
2	$f''(x) = -\sin x$	$-\frac{1}{2}$	$\frac{1}{2!}\left(\frac{-1}{2}\right) = \frac{-1}{4}$



$$\boxed{\sin\frac{\pi}{6} = \frac{1}{2} \quad \Big| \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}}$$

$$\rightarrow \quad \sin x \approx \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2$$

powers of $\left(x - \frac{\pi}{6}\right)$

(6)

k	$f^{(k)}(x)$	$f^{(k)}(2)$	$\frac{f^{(k)}(2)}{k!}$
0	$f(x) = (x+2)^{-1}$	$\frac{1}{4}$	$\frac{1}{4}$
1	$f'(x) = -(x+2)^{-2}$	$\frac{-1}{16}$	$\frac{-1}{16}$
2	$f''(x) = 2(x+2)^{-3}$	$\frac{1}{32}$	$\frac{1}{2!}\left(\frac{1}{32}\right) = \frac{1}{64}$

$$\frac{1}{x+2} \approx \frac{1}{4} - \frac{1}{16}(x-2) + \frac{1}{64}(x-2)^2$$

powers of $(x-2)$