① $\int_0^1 \frac{e^{9x}}{2+4e^{9x}}\,dx$

Sub $u = 2+4e^{9x}$

$du = 36e^{9x}\,dx$

$\frac{1}{36}du = e^{9x}\,dx$

When $x=0$ $u=6$

$x=1$ $u = 2+4e^9$

$= \frac{1}{36}\int_6^{2+4e^9} \frac{du}{u}$

$= \frac{1}{36}\left[\ln|u|\right]_6^{2+4e^9}$

$= \frac{1}{36}\left[\ln(2+4e^9) - \ln 6\right]$

≈ 0.24

(2) $\int_0^{\frac{\pi}{4}} 4\sqrt{\tan x}\,\sec^2 x\,dx$

Sub $u = \tan x$

$du = \sec^2 x\,dx$

When $x=0$ $u=0$ $\tan 0 = 0$

$x=\frac{\pi}{4}$ $u=1$ $\tan\frac{\pi}{4}=1$

$= \int_0^1 4\sqrt{u}\,du$

$= \int_0^1 u^{1/4}\,du$

$= \left[\frac{4}{5}u^{5/4}\right]_0^1$

$= \frac{4}{5} - 0$

$= \frac{4}{5}$

③ $\int \frac{x}{\sqrt{4-x^2}}\,dx$

Sub $u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \int \frac{-\frac{1}{2}\,du}{\sqrt{u}}$

$= \int -\frac{1}{2}u^{-1/2}\,du$

$= -u^{1/2} + C$

$= -\sqrt{4-x^2} + C$

(4) $\int \frac{4\,dx}{\sec x \cdot e^{\sin x}}$

$= \int 4e^{-\sin x} \cos x\,dx$

Sub $u = -\sin x$

$du = -\cos x\,dx$

$-du = \cos x\,dx$ ⟵

$= -\int 4e^{u}\,du$

$= -4e^{u} + C$

$= -4e^{-\sin x} + C$

⑤ $\int \frac{x}{\sqrt{1+x}}\,dx$

Integration by Parts

	D	I
⊕	x	$(1+x)^{-1/2}$
⊖	1	$2(1+x)^{1/2}$
	0	$\frac{4}{3}(1+x)^{3/2}$

$$\int \frac{x}{\sqrt{1+x}}\,dx = 2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2} + C$$

Alternatively : $\boxed{\int u\,dv = uv - \int v\,du}$

$u = x \qquad dv = (1+x)^{-1/2}\,dx$

$du = dx \qquad v = 2(1+x)^{1/2}$

$$\int x(1+x)^{-1/2}\,dx = 2x(1+x)^{1/2} - \int 2(1+x)^{1/2}\,dx$$

$$= 2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2} + C$$

same answer as above

(6) $\int_0^{\frac{\pi}{4}} \frac{5-8\sin(2x)}{\cos^2 x}\,dx$

$= \int_0^{\frac{\pi}{4}} \left(\frac{5}{\cos^2 x} - \frac{8\sin(2x)}{\cos^2 x}\right) dx$ $\quad \frac{1}{\cos x} = \sec x$

$\sin 2x = 2\sin x \cos x$

$= \int_0^{\frac{\pi}{4}} \left(5\sec^2 x - \frac{16\sin x\cos x}{\cos^2 x}\right) dx$

$= \int_0^{\frac{\pi}{4}} \left(5\sec^2 x - 16\frac{\sin x}{\cos x}\right) dx$ $\quad \frac{\sin x}{\cos x} = \tan x$

$= \int_0^{\frac{\pi}{4}} (5\sec^2 x - 16\tan x)\,dx$

$= \left[5\tan x + 16\ln|\cos x|\right]_0^{\frac{\pi}{4}}$

$= \left(5 + 16\ln\left(\frac{1}{\sqrt{2}}\right)\right) - (0 + 16\ln 1)$ $\quad \ln 1 = 0$

$= 5 + 16\ln\left(\frac{1}{\sqrt{2}}\right)$

or using $\ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}\ln 2$

$= 5 - 8\ln 2$

(7) $\int \frac{\ln x}{x^2}\,dx$

Integration by Parts

	D	I	
(u)	$\ln x$	x^{-2}	(dv)
(du)	$\frac{1}{x}$	$-x^{-1}$	(v)
	⋮	⋮	

$$\int u\,dv = uv - \int v\,du$$

$$\int x^{-2}\ln x\,dx = -x^{-1}\ln x - \int (-x^{-2})\,dx$$

$$= -x^{-1}\ln x + \int x^{-2}\,dx$$

$$= -x^{-1}\ln x - x^{-1} + C$$

(8) $\int \sin^2 6x\,dx$ $\qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$= \frac{1}{2}\int (1-\cos 12x)\,dx$

$= \frac{1}{2}\left(x - \frac{1}{12}\sin 12x\right) + C$

or $\frac{1}{24}(12x - \sin 12x) + C$

Could sub $u = 12x$

$du = 12\,dx$

$\frac{1}{12}\,du = dx$

$\frac{1}{24}\int (1-\cos u)\,du$

$= \frac{1}{24}(u - \sin u) + C$

$= \frac{1}{24}(12x - \sin 12x) + C$

(9) $\int \frac{e^{-x}\,dx}{1+e^{-2x}}$

$(e^{-x})^2$

Sub $u = e^{-x}$

$du = -e^{-x}\,dx$

$-du = e^{-x}\,dx$

$= \int \frac{-du}{1+u^2}$

$= -\tan^{-1}u + C$

$= -\tan^{-1}(e^{-x}) + C$

(10) $\int \tan^3\theta \sec^5\theta \, d\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \int \tan\theta \cdot \tan^2\theta \cdot \sec^5\theta \, d\theta$

$= \int \tan\theta \left(\sec^2\theta - 1\right) \sec^5\theta \, d\theta$

$= \int \tan\theta \left(\sec^7\theta - \sec^5\theta\right) d\theta$

$= \int \sec\theta \tan\theta \left(\sec^6\theta - \sec^4\theta\right) d\theta$

Sub $u = \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$= \int \left(u^6 - u^4\right) du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{1}{7}\sec^7\theta - \frac{1}{5}\sec^5\theta + C$

(11) $\int \frac{5\,dx}{x^2+8x+17}$

$x^2+\boxed{8}x+17$

$8/2=4$

$= (x+4)^2 + 1$

$= \int \frac{5\,dx}{1+(x+4)^2}$ Sub $u = x+4$

$du = dx$

$= \int \frac{5\,du}{1+u^2}$

$= 5[\tan^{-1} u] + C$

$= 5\tan^{-1}(x+4) + C$