1. $v(0)=30$ $\quad a=-110$

$$v(t)=\int a(t)\,dt$$

$$v(t)=\int -110\,dt$$

$$v(t)=-110t+C$$

$v(0)=30$: $\quad 30=C$

$$v(t)=-110t+30$$

$$s(t)=\int v(t)\,dt$$

$$s(t)=\int(-110t+30)\,dt$$

$$s(t)=-55t^2+30t+C_1$$

$s(0)=0$ $\quad 0=C_1$

Set initial displacement to zero $\quad s(t)=-55t^2+30t$

Stopping time: Set $v=0$

$$0=-110t+30$$

$$110t=30$$

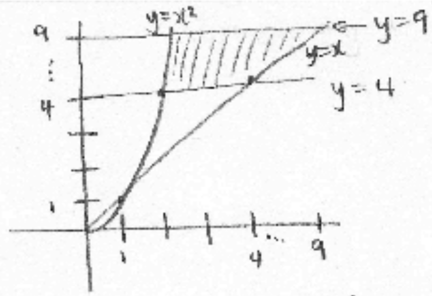
$$t=\frac{3}{11}$$

$$s(t)=-55t^2+30t$$

$$s\left(\frac{3}{11}\right)\approx 4.1\text{ m}$$

Stopping distance is approx 4.1m

(2)



Horizontal rectangles

x_r: $y = x$

$x_r = y$ →

x_l: $y = x^2$

$x = \pm\sqrt{y}$ ↗

$x_l = \sqrt{y}$

$$A = \int_4^9 (x_r - x_l)\,dy$$

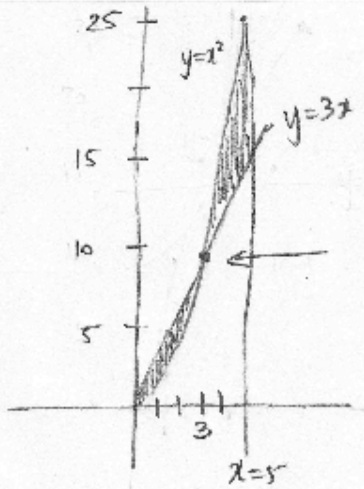
$$= \int_4^9 (y - \sqrt{y})\,dy$$

$$= \left[\frac{y^2}{2} - \frac{2}{3}y^{3/2}\right]_4^9$$

$$= \left[\frac{81}{2} - \frac{2}{3}\cdot 27\right] - \left[\frac{16}{2} - \frac{2}{3}\cdot 8\right]$$

$$= \frac{119}{6}$$

(3)



$x^2=3x$
$x^2-3x=0$
$x(x-3)=0$
$x=0,3$

$$\text{Total Area} = \int_3^3 (y_t-y_b)\,dx + \int_3^5 (y_t-y_b)\,dx$$

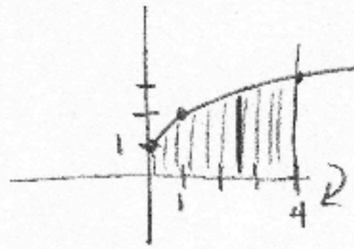
$$= \int_0^3 (3x-x^2)\,dx + \int_3^5 (x^2-3x)\,dx$$

$$= \left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3 + \left[\frac{x^3}{3} - \frac{3}{2}x^2\right]_3^5$$

$$= \underbrace{\left[\frac{3}{2}\cdot 9 - \frac{27}{3}\right]}-[0] + \left[\frac{5^3}{3} - \underbrace{\frac{3}{2}\cdot 25\right] - \left[\frac{3^3}{3}} - \frac{3}{2}\cdot 9\right]$$

$$= \frac{79}{6}$$

(4)



$y = \sqrt{x} + 1$

Disk Method

$dV = \pi \cdot \text{radius}^2 \cdot \text{thickness}$

$dV = \pi y^2 \, dx$

$V = \pi \int_0^4 y^2 \, dx$

$V = \pi \int_0^4 (\sqrt{x}+1)^2 \, dx$

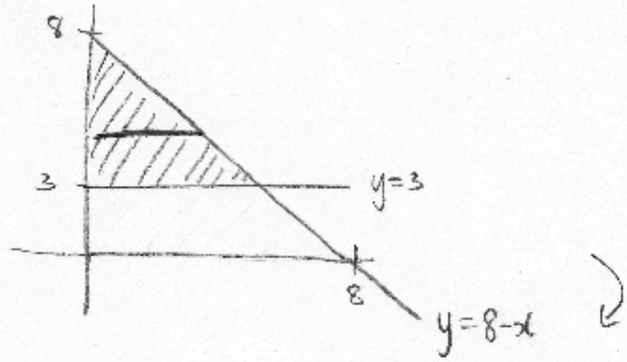
$V = \pi \int_0^4 (x + 2\sqrt{x} + 1) \, dx$

$V = \pi \left[\frac{x^2}{2} + \frac{4}{3} x^{3/2} + x \right]_0^4$

$V = \pi \left[\left(\frac{16}{2} + \frac{4}{3} \cdot 8 + 4 \right) - 0 \right]$

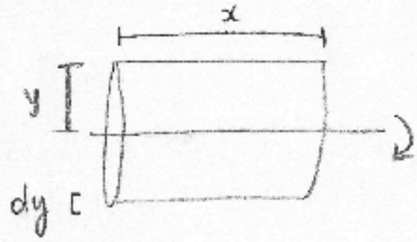
$= \frac{68\pi}{3}$

(5)



Shell Method $\quad dV = 2\pi \cdot \text{radius} \cdot \text{height} \cdot \text{thickness}$

$= 2\pi\, y\, x\, dy$



$$V = 2\pi \int_3^8 y\, x\, dy$$

$$V = 2\pi \int_3^8 y(8-y)\, dy$$

$y = 8 - x \rightarrow$
$x = 8 - y$

$$V = 2\pi \int_3^8 [8y - y^2]\, dy$$

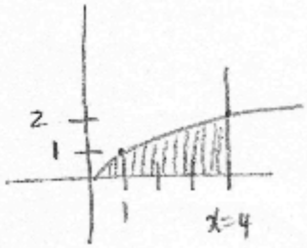
$$V = 2\pi \left[4y^2 - \frac{y^3}{3}\right]_3^8$$

$$V = 2\pi \left[\left[4 \cdot 64 - \frac{512}{3}\right] - \left[4 \cdot 9 - \frac{27}{3}\right]\right]$$

$$V = 2\pi \left[\frac{175}{3}\right]$$

$$V = \frac{350\pi}{3}$$

(6)



$$\bar{x} = \frac{\int_0^4 x(y_t - y_b)\,dx}{\int_0^4 (y_t - y_b)\,dx}$$

$$A = \int_0^4 (y_t - y_b)\,dx$$
$$= \int_0^4 \sqrt{x}\,dx$$
$$= \left[\frac{2}{3}x^{3/2}\right]_0^4$$
$$= \frac{2}{3}\cdot 8 - 0$$
$$= \frac{16}{3}$$

$$\bar{x} = \frac{1}{A}\int_0^4 x(y_t - y_b)\,dx$$
$$= \frac{1}{A}\int_0^4 x\underbrace{\sqrt{x}}_{y_t - y_b}\,dx$$
$$= \frac{1}{A}\int_0^4 x^{3/2}\,dx$$
$$= \frac{1}{A}\left[\frac{2}{5}x^{5/2}\right]_0^4$$
$$= \frac{3}{16}\left[\frac{64}{5}\right]$$
$$= \frac{12}{5}$$

$$\bar{y} = \frac{\int_0^2 y(x_r - x_l)\,dy}{\int_0^2 (x_r - x_l)\,dy}$$

We know $A = \frac{16}{3}$

$$\bar{y} = \frac{1}{A}\int_0^2 y(x_r - x_l)\,dy$$

$x_r = 4$

x_l : $y = \sqrt{x}$

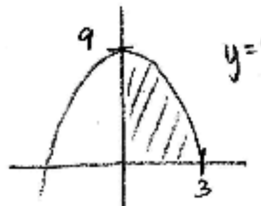
$y^2 = x$

$x_l = y^2$

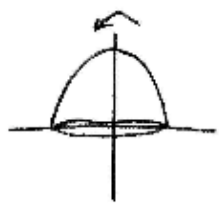
$$\bar{y} = \frac{1}{A}\int_0^2 y(4 - y^2)\,dy$$
$$= \frac{1}{A}\int_0^2 (4y - y^3)\,dy$$
$$= \frac{1}{A}\left[2y^2 - \frac{y^4}{4}\right]_0^2$$
$$= \frac{3}{16}[4]$$
$$= \frac{12}{16}$$
$$= \frac{3}{4}$$

$$(\bar{x}, \bar{y}) = \left(\frac{12}{5}, \frac{3}{4}\right)$$

⑦



$y = 9 - x^2$



$\bar{x} = 0$ by symmetry

$$\bar{y} = \frac{\int_0^9 y x^2 \, dy}{\int_0^9 x^2 \, dy}$$

$$B = \int_0^9 x^2 \, dy$$

$y = 9 - x^2 \Rightarrow$
$x^2 = 9 - y$

$$B = \int_0^9 (9-y) \, dy$$
$$= \left[9y - \frac{y^2}{2} \right]_0^9$$
$$= \left[81 - \frac{81}{2} \right] - 0$$
$$= \frac{81}{2}$$

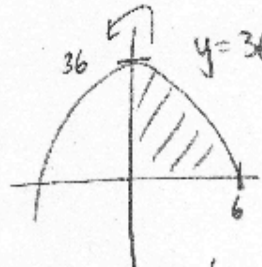
$$\bar{y} = \frac{1}{B} \int_0^9 y x^2 \, dy$$

$x^2 = 9 - y$

$$\bar{y} = \frac{1}{B} \int_0^9 y(9-y) \, dy$$
$$= \frac{1}{B} \int_0^9 (9y - y^2) \, dy$$
$$= \frac{1}{B} \left[9\frac{y^2}{2} - \frac{y^3}{3} \right]_0^9$$
$$= \frac{1}{B} \left[\left[9 \cdot \frac{81}{2} - \frac{9^3}{3} \right] - 0 \right]$$
$$= \frac{2}{81} \left[\frac{243}{2} \right]$$
$$= 3$$

$$(\bar{x}, \bar{y}) = (0, 3)$$

(8)



$$I_y = 2\pi k \int_0^6 x^3 (y_t - y_b)\,dx$$

$$= 2\pi k \int_0^6 x^3 (36 - x^2 - 0)\,dx$$

$$= 2\pi k \int_0^6 (36x^3 - x^5)\,dx$$

$$= 2\pi k \left[9x^4 - \frac{x^6}{6} \right]_0^6$$

$$= 2\pi k \left[\left(9 \cdot 6^4 - \frac{6^6}{6}\right) - 0 \right]$$

$$= 2\pi k [3888]$$

$$= 7776\pi k$$

Now $R_y = \sqrt{\frac{I_y}{m}}$

$m = k \cdot \text{volume}$

$$= k \cdot 2\pi \int_0^6 x\,y\,dx$$

[Shell Method]

$$= 2\pi k \int_0^6 x(36 - x^2)\,dx$$

$$= 2\pi k \int_0^6 (36x - x^3)\,dx$$

$$m = 2\pi k \left[18x^2 - \frac{x^4}{4} \right]_0^6$$

$$= 2\pi k \left[\left(18 \cdot 36 - \frac{6^4}{4}\right) - 0 \right]$$

$$= 648\pi k$$

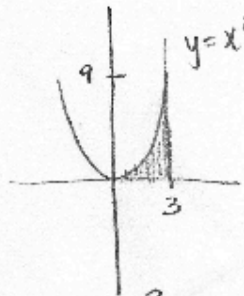
$$\frac{I_y}{m} = \frac{7776\pi k}{648\pi k}$$

$$= 12$$

$$R_y = \sqrt{12}$$

$$= 2\sqrt{3}$$

9



$$I_x = k\int_0^9 y^2(x_r - x_l)\,dy$$

$x_r = 3$

x_l: $y = x^2$

$x = \pm\sqrt{y}$

$x_l = \sqrt{y}$

$$\rightarrow I_x = k\int_0^9 y^2(3-\sqrt{y})\,dy$$

$$= k\int_0^9 (3y^2 - y^{5/2})\,dy$$

$$= k\left[y^3 - \tfrac{2}{7}y^{7/2}\right]_0^9$$

$$= k\left[\left(9^3 - \tfrac{2}{7}\cdot 9^{7/2}\right) - 0\right]$$

$$= k\left[\tfrac{729}{7}\right]$$

$$= \frac{729k}{7}$$

$$R_x = \sqrt{\frac{I_x}{m}}$$

$m = k \cdot \text{area}$

$$= k\int_0^9 (3-\sqrt{y})\,dy$$

$$= k\left[3y - \tfrac{2}{3}y^{3/2}\right]_0^9$$

$$= k\left[\left(27 - \tfrac{2}{3}\cdot 9^{3/2}\right) - 0\right]$$

$$= 9k$$

$$\frac{I_x}{m} = \frac{729k}{7}\cdot\frac{1}{9k}$$

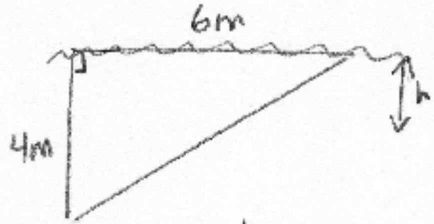
$$= \frac{81}{7}$$

$$R_x = \sqrt{\frac{81}{7}}$$

$$= \frac{9}{\sqrt{7}}$$

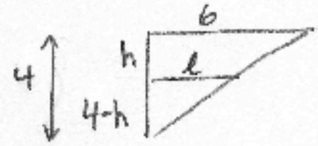
$$= \frac{9\sqrt{7}}{7}$$

(10)



$$F = 9800 \int_a^b h\,l\,dh$$

$$0 \le h \le 4$$



Similar triangles $\frac{6}{4} = \frac{l}{4-h}$

$$l = \frac{6}{4}(4-h)$$

$$l = \frac{3}{2}(4-h)$$

$$F = 9800 \int_0^4 h \cdot \frac{3}{2}(4-h)\,dh$$

$$= 9800 \cdot \frac{3}{2} \int_0^4 (4h - h^2)\,dh$$

$$= 9800 \cdot \frac{3}{2} \left[2h^2 - \frac{h^3}{3}\right]_0^4$$

$$= 9800 \cdot \frac{3}{2} \left[\left[2 \cdot 4^2 - \frac{4^3}{3}\right] - 0\right]$$

$$= 9800 \cdot \frac{3}{2} \left[\frac{32}{3}\right]$$

$$= 156{,}800 \text{ N}$$

(11) a) $F(x) = kx$

$5 = k\,0.04$

$k = \frac{5}{0.04}$

$k = 125$

$x = 0.14 - 0.1 = 0.04\,m$
is the amount stretched

b) Using $F(x) = 125x$,

x = amount stretched

$0 \leq x \leq 0.12$

$W = \int_0^{0.12} F(x)\,dx$

$= \int_0^{0.12} 125x\,dx$

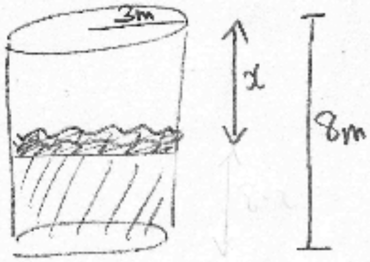
$W = 125\left[\frac{x^2}{2}\right]_0^{0.12}$

$W = 125\left[\frac{0.0144}{2} - 0\right]$

$= 0.9\ N\cdot m$

or $0.9\ J$

(12)



$4 \le x \le 8$ Tank initially half-full

A slice of water weighs $9800 \cdot \pi \cdot 3^2 dx$

Has to be moved x m

$$W = \int_4^8 9800\pi \cdot 3^2 x \, dx$$

$$= 9800 \cdot \pi \cdot 9 \int_4^8 x \, dx$$

$$= 9800 \cdot \pi \cdot 9 \left[\frac{x^2}{2}\right]_4^8$$

$$= 9800 \cdot \pi \cdot 9 \left[\frac{64}{2} - \frac{16}{2}\right]$$

$$\approx 6.7 \times 10^6 \text{ J}$$