Math 185 Quiz Four Total: 30 marks

CIVIL

Name:________________________

Show all your work for full marks.

1. [5 marks] Find $f'(0)$ for $f(x) = (2x+5)\cos^{-1}x - \frac{1}{3}\tan^{-1}(8x+1)$

$$f'(x) = (2x+5)\cdot\frac{-1}{\sqrt{1-x^2}} + (\cos^{-1}x)(2) + \frac{1}{3}\cdot\frac{1}{1+(8x+1)^2}\cdot 8$$

$$f'(0) = 5(-1) + (\cos^{-1}0)(2) + \frac{1}{3}\cdot\frac{1}{2}\cdot 8$$

$$= -5 + \frac{\pi}{2}(2) + \frac{4}{3}$$

$$= \frac{-11}{3} + \pi \quad \text{or} \quad \frac{-11+3\pi}{3}$$

2. [4 marks] Find the angle θ between 0 and $\frac{\pi}{2}$ at which the function $f(\theta) = 4\sin\theta + 7\cos\theta$ is maximized. Leave your answer in exact form (no decimals).

Maximize $f(\theta) = 4\sin\theta + 7\cos\theta$

$f'(\theta) = 4\cos\theta - 7\sin\theta$ set $= 0$

$$4\cos\theta - 7\sin\theta = 0$$

$$4\cos\theta = 7\sin\theta$$

$$\frac{4}{7} = \tan\theta$$

$$\boxed{\theta = \tan^{-1}\left(\frac{4}{7}\right)}$$

Check: $f''(\theta) = -4\sin\theta - 7\cos\theta$

$f''(30°) < 0$ $\frown$ max ✓

$\tan^{-1}\left(\frac{4}{7}\right) \approx 30°$

3. [4 marks] Find $f'(x)$ for

$$f(x) = 5^{4x^2-3x}$$

$$f'(x) = \ln 5 \cdot 5^{4x^2-3x}(8x-3)$$

$$\text{or } \ln 5 \cdot (8x-3) \cdot 5^{4x^2-3x}$$

4. [4 marks] Find $f'(x)$ for $f(x) = \ln\sqrt{\frac{x^2+3}{5x-4}}$. Simplify your answer.

$$f(x) = \frac{1}{2}\ln\left(\frac{x^2+3}{5x-4}\right)$$

$$f(x) = \frac{1}{2}\left[\ln(x^2+3) - \ln(5x-4)\right]$$

$$f'(x) = \frac{1}{2}\left[\frac{2x}{x^2+3} - \frac{5}{5x-4}\right]$$

$$= \frac{1}{2}\left[\frac{2x(5x-4) - 5(x^2+3)}{(x^2+3)(5x-4)}\right] \checkmark$$

$$= \frac{5x^2-8x-15}{2(x^2+3)(5x-4)}$$

5. [6 marks] Evaluate the following integrals:

a) $\int(\sqrt{x} + \frac{1}{x^3})dx$

$= \int(x^{1/2} + x^{-3})dx$

$= \frac{2}{3}x^{3/2} - \frac{1}{2}x^{-2} + C$

b) $\int \sqrt[5]{r}(r^6 - r^8)dr$

$= \int r^{1/5}(r^6 - r^8)dr$

$= \int(r^{31/5} - r^{41/5})dr$

$= \frac{5}{36}r^{36/5} - \frac{5}{46}r^{46/5} + C$

6. [4 marks] Evaluate $\int \frac{2x^2}{\sqrt{2x^3+1}}dx$.

Let $u = 2x^3+1$

$du = 6x^2\,dx$

$2x^2\,dx = \frac{1}{3}du$

Integral $= \int \frac{1}{3} \cdot \frac{1}{\sqrt{u}}\,du$

$= \frac{1}{3}(2u^{1/2}) + C$

$= \frac{2}{3}\sqrt{2x^3+1} + C$

7. [3 marks] Find $f(x)$ if $f(-1) = 6$ and $f'(x) = x^2 + x^{-2}$.

$$f(x) = \int (x^2 + x^{-2})\,dx$$

$$f(x) = \frac{1}{3}x^3 - x^{-1} + C$$

Sub
$x = -1$
$f(x) = 6$

$$6 = \frac{1}{3}(-1)^3 - (-1)^{-1} + C$$

$$6 = -\frac{1}{3} + 1 + C$$

$$\frac{18}{3} = \frac{2}{3} + C$$

$$C = \frac{16}{3}$$

$$\boxed{f(x) = \frac{1}{3}x^3 - x^{-1} + \frac{16}{3}}$$

Math 185 Quiz Four Total: 30 marks

MECH

Name:____________________________________

Show all your work for full marks.

1. [5 marks] Find $f'(0)$ for $f(x) = \frac{1}{9}\tan^{-1}(4x+1) + (7-12x)\cos^{-1}x$

$$f'(x) = \frac{1}{9} \cdot \frac{1}{1+(4x+1)^2} \cdot (4) + (7-12x)\frac{-1}{\sqrt{1-x^2}} + \cos^{-1}x \cdot (-12)$$

$$f'(0) = \frac{1}{9} \cdot \frac{1}{2}(4) + 7(-1) + \cos^{-1}(0)(-12)$$

$$= \frac{2}{9} - 7 \quad -12\left(\frac{\pi}{2}\right)$$

$$= \frac{-61}{9} - 6\pi$$

$$\text{or } \frac{-(61+54\pi)}{9}$$

2. [4 marks] Find the angle θ between 0 and $\frac{\pi}{2}$ at which the function $f(\theta) = \theta + 3\cos\theta$ is maximized. Leave your answer in exact form (no decimals).

Maximize $f(\theta) = \theta + 3\cos\theta$

$f'(\theta) = 1 - 3\sin\theta$ Set $= 0$

$$1 - 3\sin\theta = 0$$

$$1 = 3\sin\theta$$

$$\frac{1}{3} = \sin\theta$$

$$\boxed{\theta = \sin^{-1}\left(\frac{1}{3}\right)}$$

Check: $f''(\theta) = -3\cos\theta$

$f''(19°) < 0$ $\cap$ max ✓

$\sin^{-1}\left(\frac{1}{3}\right) \approx 19°$

(3.) [4 marks] Find $f'(x)$ for [redacted]

$$f(x) = 2^{7x^2-5x}$$

$$f'(x) = \ln 2 \cdot 2^{7x^2-5x} \cdot (14x-5)$$

$$\text{or } \ln 2 \cdot (14x-5) \cdot 2^{7x^2-5x}$$

(4.) [4 marks] Find $f'(x)$ for $f(x) = \ln \sqrt[3]{\frac{x^2-10}{6x-9}}$. Simplify your answer.

$$f(x) = \frac{1}{3} \cdot \ln\left(\frac{x^2-10}{6x-9}\right)$$

$$= \frac{1}{3}\left[\ln(x^2-10) - \ln(6x-9)\right]$$

$$= \frac{1}{3}\left[\frac{2x}{x^2-10} - \frac{6}{6x-9}\right]$$

$$= \frac{1}{3}\left[\frac{2x(6x-9) - 6(x^2-10)}{(x^2-10)(6x-9)}\right] \checkmark$$

$$= \frac{6x^2-18x+60}{3(x^2-10)(6x-9)}$$

$$\text{or } \frac{2x^2-6x+20}{(x^2-10)(6x-9)}$$

5. [6 marks] Evaluate the following integrals:

a) $\int(\frac{1}{x^5} + \sqrt[4]{x})dx$

$$= \int(x^{-5} + x^{1/4})dx$$

$$= \frac{-1}{4}x^{-4} + \frac{4}{5}x^{5/4} + C$$

b) $\int \sqrt[3]{s}(s^4 - s^7)ds$

$$= \int s^{1/3}(s^4 - s^7)ds$$

$$= \int(s^{13/3} - s^{22/3})ds$$

$$= \frac{3}{16}s^{16/3} - \frac{3}{25}s^{25/3} + C$$

6. [3 marks] Find y if $\frac{dy}{dx} = x^3 + x^{-3}$ and the point $(-2, 7)$ lies on the curve.

$$y = \int(x^3 + x^{-3})dx$$

$$y = \frac{1}{4}x^4 - \frac{1}{2}x^{-2} + C$$

Sub $x = -2$, $y = 7$

$$7 = \frac{1}{4}(-2)^4 - \frac{1}{2}(-2)^{-2} + C$$

$$7 = 4 - \frac{1}{8} + C$$

$$\frac{56}{8} = \frac{31}{8} + C \qquad C = \frac{25}{8}$$

$$\boxed{y = \frac{1}{4}x^4 - \frac{1}{2}x^{-2} + \frac{25}{8}}$$

7. [4 marks] Evaluate $\int \frac{3x^3}{\sqrt{3x^4-1}} dx$.

Let $u = 3x^4 - 1$

$du = 12x^3 \, dx$

$3x^3 dx = \frac{1}{4} du$

$$\text{Integral} = \int \frac{1}{4} \cdot \frac{1}{\sqrt{u}} \, du$$

$$= \frac{1}{4}\left(2u^{1/2}\right) + C$$

$$= \frac{1}{2}\sqrt{3x^4-1} + C$$