1. [6 marks] Find $f'(x)$:

a) $f(x) = 3\tan(\pi x)$

$$f'(x) = 3\sec^2(\pi x) \cdot \pi$$

$$= 3\pi \sec^2(\pi x)$$

b) $f(x) = 8\csc^4 x$

Rewrite $f(x) = 8[\csc x]^4$

$$f'(x) = 32\csc^3 x\,(-\csc x \cot x)$$

$$= -32\csc^4 x \cot x$$

c) $f(x) = 2x^3 \sin^{-1} x$

$$f'(x) = 2x^3 \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\,(6x^2)$$

$$= \frac{2x^3}{\sqrt{1-x^2}} + 6x^2 \sin^{-1} x$$

2. [6 marks] Find $\frac{dy}{dx}$:

a) $y = \log_3(5 - x^2)$

$$\frac{dy}{dx} = \frac{1}{\ln 3} \cdot \frac{1}{5-x^2} (-2x)$$

$$= \frac{-2x}{(\ln 3)(5-x^2)}$$

b) $y = 10^{3x+8}$

$$\frac{dy}{dx} = (\ln 10)(10^{3x+8})(3)$$

$$= (3\ln 10)\, 10^{3x+8}$$

c) $y = e^{7x} \ln(x^6 + 1)$

$$\frac{dy}{dx} = e^{7x}\left[\frac{1}{x^6+1}(6x^5)\right] + \ln(x^6+1)(e^{7x} \cdot 7)$$

$$= e^{7x} \cdot \frac{6x^5}{x^6+1} + 7e^{7x} \ln(x^6+1)$$

3. [2 marks] Find $f'(x)$ for $f(x) = \ln(\sin(x^4))$.

$$f'(x) = \frac{1}{\sin(x^4)} \frac{d}{dx}[\sin(x^4)]$$

$$= \frac{1}{\sin(x^4)} \cdot [\cos(x^4)(4x^3)]$$

$$= 4x^3 \cot(x^4)$$

4. [4 marks] Find the linearization of $f(x) = \sqrt[4]{x}$ at $a = 16$. Simplify the coefficients.

$f(x) = x^{1/4}$

$f(16) = 2$

$f'(x) = \frac{1}{4} x^{-3/4}$

$f'(16) = \frac{1}{4} (16)^{-3/4}$

$= \frac{1}{4} \cdot \frac{1}{16^{3/4}}$

$= \frac{1}{4} \cdot \frac{1}{8}$

$= \frac{1}{32}$

$f(x) \approx f(a) + f'(a) \cdot (x-a)$

$$\boxed{x^{1/4} \approx 2 + \frac{1}{32}(x-16)}$$

(valid near $x=16$)

5. [3 marks] Given $f = 0.9r^{-2}$, find $\frac{df}{f}$ if $\frac{dr}{r} = 0.04$. Show all your work.

$\frac{df}{dr} = -1.8r^{-3}$

$df = -1.8r^{-3} dr$

$\frac{df}{f} = \frac{-1.8r^{-3} dr}{0.9r^{-2}}$

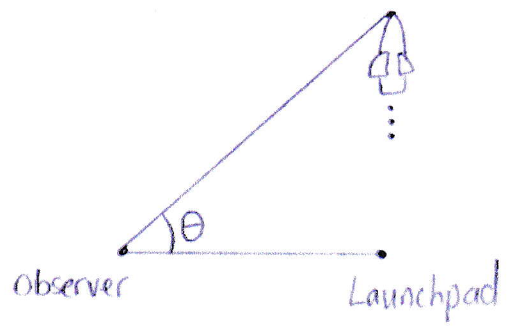
$\frac{df}{f} = -2r^{-1} dr$

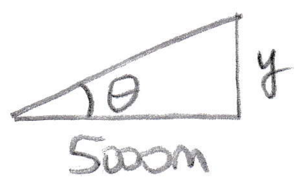
$\frac{df}{f} = -2\left(\frac{dr}{r}\right)$

$\frac{df}{f} = -2(0.04)$

$\frac{df}{f} = -0.08$

6. [4 marks] A rocket takes off vertically and rises at 800 m/s. The rocket's velocity remains constant as it rises. If an observer is 5,000 m from the launchpad, how fast is θ changing when the rocket is 2,000 m above the ground? Round your answer to two decimal places.





$\frac{dy}{dt} = 800$ m/s

? $\frac{d\theta}{dt}$ when $y = 2000$m

$$\tan\theta = \frac{y}{5000}$$

$$\theta = \tan^{-1}\left(\frac{y}{5000}\right)$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dy} \cdot \frac{dy}{dt}$$

$$= \left[\frac{1}{1+\left(\frac{y}{5000}\right)^2} \cdot \frac{1}{5000}\right] \cdot \frac{dy}{dt}$$

Now sub $y = 2000$, $\frac{dy}{dt} = 800$:

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{2000}{5000}\right)^2} \cdot \frac{1}{5000} \cdot 800$$

≈ 0.14 rad/s

or $7.90°$/s