1. [2 marks] We want to solve $x^6 - 7x - 40 = 0$ using Newton's Method with $x_0 = 2$. Fill in the following table to find x_1. You do **not** need to find x_2. Round x_1 to two decimal places.

$f(x) = x^6 - 7x - 40$

$f'(x) = 6x^5 - 7$

x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
$x_0 = 2$	10	185	1.95

2

2. [3 marks] Find the equation of the tangent line to $y = \frac{2}{3}(x^4 - 2)^5$ at the point $(1, \frac{-2}{3})$. Write your answer in slope-intercept form.

$y = \frac{2}{3}(x^4 - 2)^5$

$y' = \frac{10}{3}(x^4 - 2)^4(4x^3)$ 1

$y'|_{x=1} = \frac{10}{3}(-1)^4(4)$

$y'|_{x=1} = \frac{40}{3}$ 1

$y - y_1 = m(x - x_1)$ 1

$y + \frac{2}{3} = \frac{40}{3}(x - 1)$

$y + \frac{2}{3} = \frac{40}{3}x - \frac{40}{3}$

$\boxed{y = \frac{40}{3}x - \frac{42}{3}}$

or $\boxed{y = \frac{40}{3}x - 14}$

3. [5 marks] An object's position (in metres) after t seconds is described by: $x = 50t^{-3} + 2$, $y = 8t^2 + 4t$. Find the object's velocity at $t = 2$ seconds. Round your values to one decimal place.

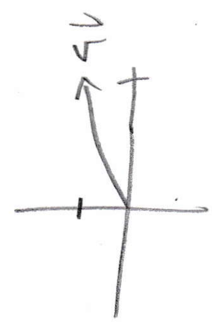
$x = 50t^{-3} + 2 \qquad y = 8t^2 + 4t$

$v_x = -150t^{-4} \qquad v_y = 16t + 4$ 2

$@t=2: \; v_x = -9.375 \qquad v_y = 36$ 1

speed $v = \sqrt{(-9.375)^2 + 36^2}$

≈ 37.2 m/s 1



$\theta = \tan^{-1}\left(\frac{36}{-9.375}\right)$ (+180°?)

$\theta \approx 104.6°$ 1

$\boxed{37.2 \text{ m/s at } 104.6°}$

4. [2 marks] A spherical snowball is melting such that its radius is decreasing by 0.02 cm per minute. How fast is the volume of the snowball changing when its radius is 9 cm? Use $V = \frac{4}{3}\pi r^3$.

$V = \frac{4}{3}\pi r^3 \qquad \frac{dr}{dt} = -0.02 \text{ cm/min} \qquad r = 9\text{cm}$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$= 4\pi r^2 \cdot \frac{dr}{dt}$ 1

$= 4\pi(9)^2(-0.02)$

$= -6.48\pi \text{ cm}^3/\text{min}$ 1

or $-20 \text{ cm}^3/\text{min}$

-0.5 if you forgot the negative sign

5. [5 marks] Find $\frac{dy}{dx}$ given $19x^3 - 5y^3 = 6x^2y$.

$$57x^2 - 15y^2 \cdot \frac{dy}{dx} = 6x^2\frac{dy}{dx} + y(12x) \quad 3$$

$$-6x^2\frac{dy}{dx} - 15y^2\frac{dy}{dx} = 12xy - 57x^2$$

$$(-6x^2 - 15y^2)\frac{dy}{dx} = 12xy - 57x^2 \quad 1$$

$$\frac{dy}{dx} = \frac{12xy - 57x^2}{-6x^2 - 15y^2} \quad 1$$

$$\text{or} \quad \frac{57x^2 - 12xy}{6x^2 + 15y^2}$$

$$\text{or} \quad \frac{19x^2 - 4xy}{2x^2 + 5y^2}$$

6. [4 marks] Given $f(x) = 2x^5 + 10x^4 - 300$:

a) Find all relative maximum and relative minimum points. Be sure to include the y-coordinates.

$f'(x) = 10x^4 + 40x^3$ 1

$10x^4 + 40x^3 = 0$

$10x^3(x+4) = 0$

$x = 0 \quad x = -4$ 1

$f'(x)$: $\oplus$ -4 $\ominus$ 0 $\oplus$

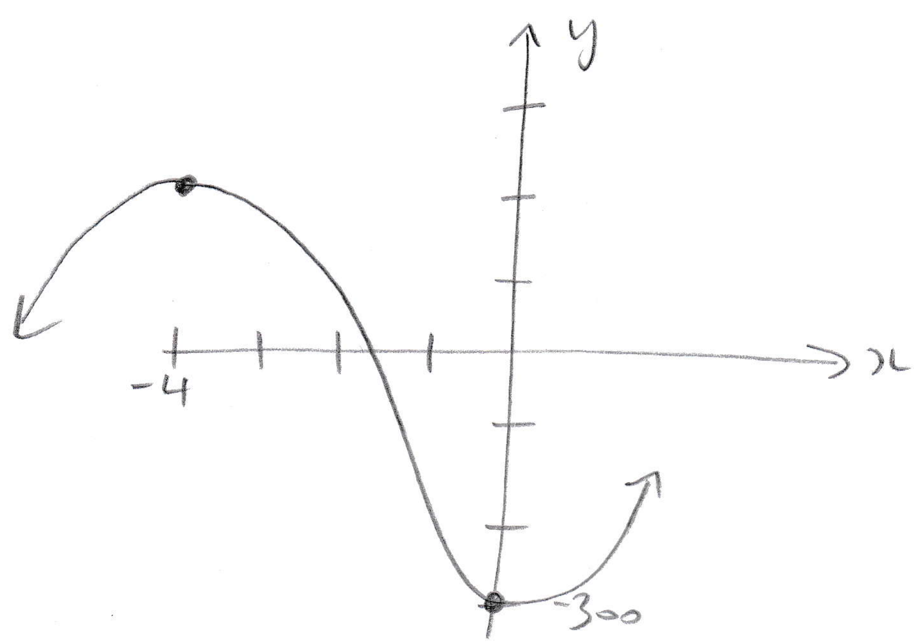
$f(x)$: INC DEC INC

relative maximum at $(-4, 212)$

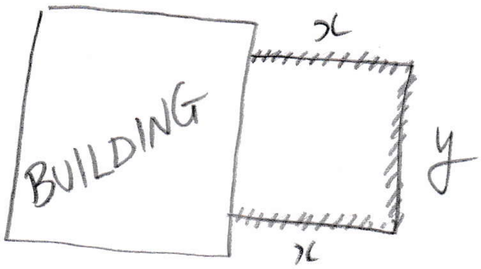
relative minimum at $(0, -300)$

b) Based on part a), make a rough sketch of the function.

1



7. [4 marks] A rectangular storage area is to be built along the side of a building. A fence is required along the remaining three sides. What is the maximum area that can be enclosed with 76m of fencing?



Maximize $A = xy$ $\qquad$ $2x + y = 76$

$A = x(76 - 2x) \leftarrow y = 76 - 2x$ 1

$A = 76x - 2x^2$

$A' = 76 - 4x$ 1

$76 - 4x = 0$

$76 = 4x$

$19 = x$ 1

$x = 19$ corresponds to $y = 76 - 2(19) = 38$

The maximum area that can be enclosed is $xy = 722\text{ m}^2$. 1