1. [4 marks] An object's displacement (in metres) is given by $s(t) = 3.8t^3 - 2.8t^2 + 3.1$, where t is measured in seconds. Find the object's acceleration at $t = 2.1$ seconds. Include the correct units.

velocity $v(t) = 11.4t^2 - 5.6t$ (1)

acceleration $a(t) = 22.8t - 5.6$ (1)

$a(2.1) = 42.28 \text{ m/s}^2$ (2)

2. [2 marks] Evaluate the following limit. Show all your work for full marks.

$\lim\limits_{x \to \infty} \frac{7-5x^2}{2x^2+3x}$

$$= \lim_{x \to \infty} \frac{\frac{7}{x^2} - \frac{5x^2}{x^2}}{\frac{2x^2}{x^2} + \frac{3x}{x^2}}$$ (1)

$$= \lim_{x \to \infty} \frac{\frac{7}{x^2} \to 0 \; - 5}{2 + \frac{3}{x} \to 0}$$

$$= \frac{0-5}{2+0}$$

$$= \frac{-5}{2}$$ (1)

3. [5 marks] Use the **limit definition** to find $f'(x)$ for $f(x) = 8 + \frac{4}{x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[8 + \frac{4}{x+h} - \left(8 + \frac{4}{x}\right)\right]$$ 2

$$= \lim_{h \to 0} \frac{1}{h}\left[8 + \frac{4}{x+h} - 8 - \frac{4}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{4}{x+h} - \frac{4}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{4x - 4(x+h)}{(x+h)x}\right]$$ 1

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{4x - 4x - 4h}{(x+h)x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{-4h}{(x+h)x}\right]$$

$$= \lim_{h \to 0} \frac{-4}{(x+h)x}$$ 1

$$= \frac{-4}{x^2}$$ 1

4. [5 marks] a) Find $f'(x)$ for $f(x) = \frac{5x^2-1}{5-3x}$. Simplify your answer.

$$f'(x) = \frac{(5-3x)(10x) - (5x^2-1)(-3)}{(5-3x)^2}$$

3

$$= \frac{50x - 30x^2 + 3(5x^2-1)}{(5-3x)^2}$$

$$= \frac{50x - 30x^2 + 15x^2 - 3}{(5-3x)^2}$$

$$= \frac{-15x^2 + 50x - 3}{(5-3x)^2}$$

1

b) Based on part a), for what values of x is $f(x)$ differentiable? (where is $f'(x)$ defined?)

$$(5-3x)^2 \neq 0$$

$$5-3x \neq 0$$

$$5 \neq 3x$$

$$\frac{5}{3} \neq x$$

All real numbers, except $x = \frac{5}{3}$

1

5. [4 marks] Find $\frac{dy}{dx}$ for each function below. Simplify your answer.

a) $y = \frac{-2}{(6x+1)^5}$

$$y = -2(6x+1)^{-5}$$

$$\frac{dy}{dx} = 10(6x+1)^{-6}(6)$$

$$= \frac{60}{(6x+1)^6}$$

2

b) $y = 3\sqrt{8x^4+9}$

$$y = 3(8x^4+9)^{1/2}$$

$$\frac{dy}{dx} = \frac{3}{2}(8x^4+9)^{-1/2}(32x^3)$$

$$= \frac{48x^3}{\sqrt{8x^4+9}}$$

2

6. [5 marks] Given $\vec{U} = \vec{\imath} - 2\vec{\jmath} + 4\vec{k}$ and $\vec{V} = \vec{\imath} + \vec{\jmath} - \vec{k}$, find:

a) $\text{proj}_{\vec{U}}(\vec{V})$

$$= \frac{\vec{u} \cdot \vec{V}}{u^2}\vec{u}$$

$$\vec{u} \cdot \vec{V} = 1(1) + (-2)(1) + 4(-1) = -5$$

$$u = \sqrt{1^2 + (-2)^2 + 4^2} = \sqrt{21}$$

$$u^2 = 21$$

3

$$= \frac{-5}{21}(\vec{\imath} - 2\vec{\jmath} + 4\vec{k})$$

or $\frac{-5}{21}\vec{\imath} + \frac{10}{21}\vec{\jmath} - \frac{20}{21}\vec{k}$

b) $\vec{U} \times \vec{V}$

$$= -2\vec{\imath} + 5\vec{\jmath} + 3\vec{k}$$

2

$$\begin{array}{ccccc} & & - & - & - \\ \vec{\imath} & \vec{\jmath} & \vec{k} & \vec{\imath} & \vec{\jmath} \\ 1 & -2 & 4 & 1 & -2 \\ 1 & 1 & -1 & 1 & 1 \end{array}$$

$$2\vec{k} \quad -4\vec{\imath} \quad \vec{\jmath} \quad 2\vec{\imath} \quad 4\vec{\jmath} \quad \vec{k}$$