(1) $\lim_{x \to -8} \frac{x^2+5x-24}{5x+40}$

$= \lim_{x \to -8} \frac{\cancel{(x+8)}(x-3)}{5\cancel{(x+8)}}$

$= \lim_{x \to -8} \frac{x-3}{5}$

$= \frac{-11}{5}$

(2) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x+2h+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2x+2h+1} + \sqrt{2x+1}}{\sqrt{2x+2h+1} + \sqrt{2x+1}}$$

Conjugate radical

$$= \lim_{h \to 0} \frac{(2x+2h+1) - (2x+1)}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h \to 0} \frac{2x+2h+1-2x-1}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{2x+2h+1} + \sqrt{2x+1}}$$

$$= \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}}$$ (evaluating at $h=0$)

$$= \frac{2}{2\sqrt{2x+1}}$$

$$= \frac{1}{\sqrt{2x+1}}$$

3 $y = (2x+1)^{2/3}(x^3-3x^2)$

$$y' = (2x+1)^{2/3}\frac{d}{dx}(x^3-3x^2) + (x^3-3x^2)\frac{d}{dx}(2x+1)^{2/3}$$

$$= (2x+1)^{2/3}(3x^2-6x) + (x^3-3x^2)\left[\frac{2}{3}(2x+1)^{-1/3}(2)\right]$$

$$y'\big|_{x=2} = 5^{2/3}(0) + (-4)\left[\frac{2}{3}(5)^{-1/3}(2)\right]$$

$$= \frac{-16}{3\cdot 5^{1/3}}$$

$$= \frac{-16}{3\cdot 5^{1/3}} \cdot \frac{5^{2/3}}{5^{2/3}}$$ ← rationalize the denominator

$$= \frac{-16\cdot 5^{2/3}}{3\cdot 5}$$

$$= \frac{-16\cdot 5^{2/3}}{15}$$

(4) $y = \dfrac{8x^2+3}{5x+1}$

$$\frac{dy}{dx} = \frac{(5x+1)\frac{d}{dx}(8x^2+3) - (8x^2+3)\frac{d}{dx}(5x+1)}{(5x+1)^2}$$

$$= \frac{(5x+1)(16x) - (8x^2+3)(5)}{(5x+1)^2}$$

$$= \frac{80x^2+16x - 5(8x^2+3)}{(5x+1)^2}$$

$$= \frac{80x^2+16x-40x^2-15}{(5x+1)^2}$$

$$= \frac{40x^2+16x-15}{(5x+1)^2}$$

(5) $\cos(xy) - \sin(3y) = 1 + x^3$

Implicit Differentiation

$$-\sin(xy) \cdot \frac{d}{dx}(xy) - \cos(3y) \cdot \frac{d}{dx}(3y) = 3x^2$$

$$-\sin(xy)\left[x\frac{dy}{dx} + y(1)\right] - \cos(3y)\left[3\frac{dy}{dx}\right] = 3x^2$$

$$-x\sin(xy)\frac{dy}{dx} - y\sin(xy) - 3\cos(3y)\frac{dy}{dx} = 3x^2$$

$$-x\sin(xy)\frac{dy}{dx} - 3\cos(3y)\frac{dy}{dx} = y\sin(xy) + 3x^2$$

$$\left[-x\sin(xy) - 3\cos(3y)\right]\frac{dy}{dx} = y\sin(xy) + 3x^2$$

$$\frac{dy}{dx} = \frac{y\sin(xy) + 3x^2}{-x\sin(xy) - 3\cos(3y)}$$

or (factoring -1 from denominator):

$$\frac{dy}{dx} = -\frac{y\sin(xy) + 3x^2}{x\sin(xy) + 3\cos(3y)}$$

(6) $y = \ln[x^3(x^2+4)]$

Recall $\ln(ab) = \ln a + \ln b$

$y = \ln x^3 + \ln(x^2+4)$

Recall $\ln(x^r) = r \ln x$

$y = 3\ln x + \ln(x^2+4)$

$y' = 3\left[\frac{1}{\ln e} \cdot \frac{1}{x}(1)\right] + \left[\frac{1}{\ln e} \cdot \frac{1}{x^2+4}(2x)\right]$

$y' = \frac{3}{x} + \frac{2x}{x^2+4}$

@ $x=1$ $\quad y' = \frac{3}{1} + \frac{2(1)}{1^2+4} = 3 + \frac{2}{5} = \frac{17}{5}$

$\boxed{m_{tan} = \frac{17}{5}}$ $\quad$ @ $x=1$ $\quad y = \ln[1^3(1^2+4)] = \ln 5$

$\boxed{(x,y) = (1, \ln 5)}$

$y - y_1 = m(x - x_1)$

$y - \ln 5 = \frac{17}{5}(x-1)$

$y - \ln 5 = \frac{17}{5}x - \frac{17}{5}$

$$\left.\begin{aligned} &y = \frac{17}{5}x - \frac{17}{5} + \ln 5 \\ \underline{\text{or}}\quad &y = \frac{17}{5}x + \frac{5\ln 5 - 17}{5} \end{aligned}\right\} \begin{aligned} &y = mx + b \\ &\text{slope-intercept form} \end{aligned}$$

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Given $e^{x} = \cos x + 1$,
find x_1 using $x_0 = -3$

→ Always use $f(x) = 0$ for Newton's Method.

$$\underbrace{e^{x} - \cos x - 1}_{f(x)} = 0$$

$f(x) = e^{x} - \cos x - 1$
$f'(x) = \ln e \cdot e^{x} \cdot 1 + \sin x$
$= e^{x} + \sin x$

x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
-3	0.0398	-0.0913	-2.5641
			$\boxed{x_1 \approx -2.56}$

Radian Mode!
Carry 2 extra 2 decimal places.

(8) $x = e^{-t^2+8t}$ $\quad y = te^{7t}$

$$v_x = \ln e \cdot e^{-t^2+8t}(-2t+8) \qquad v_y = t(\ln e \cdot e^{7t} \cdot 7) + e^{7t}(1)$$

$$= e^{-t^2+8t}(-2t+8) \qquad = 7te^{7t} + e^{7t}$$

$$= (7t+1)e^{7t}$$

@ $t = 0.2$: $v_x = 7.6\, e^{1.56}$ $\qquad v_y = 2.4e^{1.4}$

≈ 36.167 $\qquad \approx 9.732$

$\boxed{\text{Carry 2 extra decimal places}}$

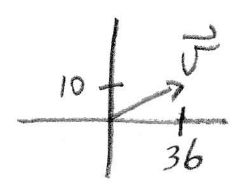
Speed $v = \sqrt{v_x^2 + v_y^2}$

≈ 37.5 m/s

Check quadrant with a rough sketch

Direction $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ $(+180°?)$

$= \tan^{-1}\left(\frac{9.732}{36.167}\right)$ $(+180°?)$



$\approx 15.1°$

$\boxed{37.5 \text{ m/s at } 15.1°}$

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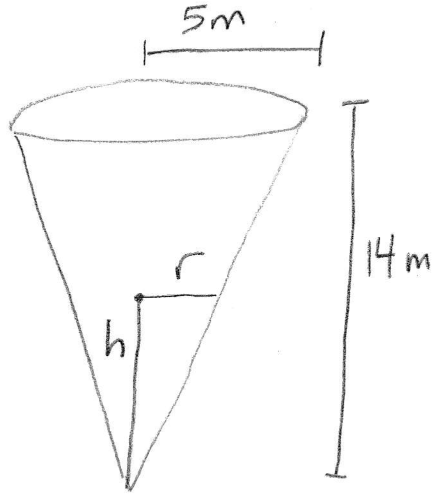


$$\frac{dV}{dt} = -2\,m^3/h$$

? $\frac{dh}{dt}$ when $h = 6\,m$?

Volume of Cone $V = \frac{1}{3}\pi r^2 h$

Similar triangles:

$$\frac{r}{h} = \frac{5}{14}$$

$$r = \frac{5}{14}h$$

$$V = \frac{1}{3}\pi\left(\frac{5}{14}h\right)^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{5}{14}\right)^2 h^3$$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(\frac{5}{14}\right)^2(3h^2)\frac{dh}{dt}$$

Sub $\frac{dV}{dt} = -2$, $h = 6$:

$$-2 = \cancel{\tfrac{1}{3}}\pi\left(\frac{5}{14}\right)^2\cancel{(3)}(36)\frac{dh}{dt}$$

$$\frac{-2\cdot 14^2}{5^2\cdot 36\pi} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-392}{900\pi}$$

or $\frac{dh}{dt} = \frac{-196}{450\pi}\,m/h$

(Negative because height is decreasing)

(10) a) x-intercepts:

set $y=0$

$0 = x^8 - 4x^6$

$0 = x^6(x^2-4)$

$0 = x^6(x-2)(x+2)$

$x = 0, 2, -2$

$\boxed{(0,0),\ (2,0),\ (-2,0)}$

y-intercept:

set $x=0$

$y = 0^8 - 4(0)^6 = 0$

$\boxed{(0,0)}$

b) $f'(x) = 8x^7 - 24x^5$

$8x^7 - 24x^5 = 0$

$8x^5(x^2-3) = 0$

$x = 0$ $\quad$ $x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

		$-\sqrt{3}$		0		$\sqrt{3}$	
$f'(x)$	$-$		$+$		$-$		$+$
$f(x)$	DEC		INC		DEC		INC

$f(-\sqrt{3})$

Relative minimum at $(-\sqrt{3}, -27)$

Relative minimum at $(\sqrt{3}, -27)$

$f(\sqrt{3})$

Relative maximum at $(0, 0)$

(11) a) $\lim_{x \to \pm\infty} \frac{x^2 - 5x}{x+2}$

$= \lim_{x \to \pm\infty} \frac{\frac{x^2}{x} - \frac{5x}{x}}{\frac{x}{x} + \frac{2}{x}}$

$= \lim_{x \to \pm\infty} \frac{x - 5}{1 + \frac{2}{x}}$ $\frac{2}{x} \to 0$

$= \lim_{x \to \pm\infty} x - 5$

$y = x - 5$ is a slant asymptote
(No horizontal asymptote)

b) $f'(x) = \frac{(x+2)(2x-5) - (x^2-5x)(1)}{(x+2)^2}$

$= \frac{2x^2 - 5x + 4x - 10 - x^2 + 5x}{(x+2)^2}$

$= \frac{x^2 + 4x - 10}{(x+2)^2}$

Critical points : $f'(x) = 0$ or undefined

$\swarrow$ $\downarrow$

$x^2 + 4x - 10 = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-10)}}{2}$

$= \frac{-4 \pm \cancel{\sqrt{56}}\, 2\sqrt{14}}{2}$

$= -2 \pm \sqrt{14}$

$(x+2)^2 = 0$

$x + 2 = 0$

$x = -2$

$f'(x)$	$+$	$-2-\sqrt{14}$	$-$	-2	$-$	$-2+\sqrt{14}$	$+$
$f(x)$	INC		DEC		DEC		INC

$f(-5.7)$

Rounding coordniates to 1 decimal place:

Relative maximum at $(-5.7, -16.5)$

Relative minimum at $(1.7, -1.5)$

$f(1.7)$

(12) Maximize $V = \pi r^2 h$

Given $r + h = 30$ $\rightarrow$ $V = \pi r^2(30 - r)$

$h = 30 - r$

$V = 30\pi r^2 - \pi r^3$

$V'(r) = 60\pi r - 3\pi r^2$

$60\pi r - 3\pi r^2 = 0$

$3\pi r(20 - r) = 0$

$r = 0, 20$

$r = 20$ $h = 30 - 20 = 10$

<u>Optional</u>: To check that $r = 20$ is indeed a maximum:

$V''(r) = 60\pi - 6\pi r$

$V''(20) < 0$

$\cap$ max ✓

The maximum volume of the cylinder is $V = \pi r^2 h = 4000\pi \text{ cm}^3$.

(13) Use $f(x) \approx f(a) + f'(a)\cdot(x-a)$

$f(x) = \sin x$

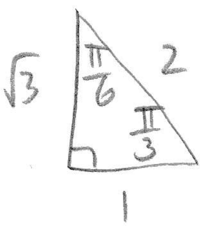
a should be near $\frac{5\pi}{18}$ with $f(a)$ exact

Choose $a = \frac{6\pi}{18} = \frac{\pi}{3}$

$f(\frac{\pi}{3}) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$f'(x) = \cos x$

$f'(\frac{\pi}{3}) = \frac{1}{2}$



$$\sin x \approx \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right)$$

At $x = \frac{5\pi}{18}$: $\sin\frac{5\pi}{18} \approx \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{5\pi}{18} - \frac{\pi}{3}\right)$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{5\pi}{18} - \frac{6\pi}{18}\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{-\pi}{18}\right)$$

$$= \frac{18\sqrt{3}}{36} - \frac{\pi}{36}$$

$$= \frac{18\sqrt{3} - \pi}{36}$$

(14)

$$f(x) = [\csc(2x)]^2 + \tan^{-1}(5x)$$

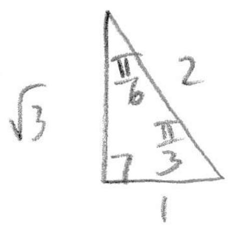
$$f'(x) = 2\csc(2x)\frac{d}{dx}\csc(2x) + \frac{1}{1+(5x)^2}\cdot 5$$

$$= 2\csc(2x)\left[-\csc(2x)\cot(2x)\cdot 2\right] + \frac{5}{1+(5x)^2}$$

$$= -4\csc^2(2x)\cot(2x) + \frac{5}{1+(5x)^2}$$

$$f'\left(\frac{\pi}{6}\right) = -4\csc^2\left(\frac{\pi}{3}\right)\cot\left(\frac{\pi}{3}\right) + \frac{5}{1+\left(\frac{5\pi}{6}\right)^2}$$

$$= -4\left(\frac{4}{3}\right)\left(\frac{1}{\sqrt{3}}\right) + \frac{5}{1+\left(\frac{5\pi}{6}\right)^2}$$



$$\approx -2.4$$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\tan\frac{\pi}{3} = \sqrt{3}$

$\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$ $\cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$

$\csc^2\frac{\pi}{3} = \frac{4}{3}$

(15) $f(x) = \log_2 (x^2 + 5x + 1) + 2^{4x}$

$$f'(x) = \frac{1}{\ln 2} \cdot \frac{1}{x^2+5x+1} \cdot (2x+5) + \ln 2 \cdot 2^{4x} \cdot 4$$

$$f'(0) = \frac{1}{\ln 2} \cdot \frac{5}{1} + \ln 2 \cdot 2^0 \cdot 4$$

$$= \frac{5}{\ln 2} + 4 \ln 2$$

or $\dfrac{5 + 4 [\ln 2]^2}{\ln 2}$

or $\dfrac{5 + 4 \ln^2 2}{\ln 2}$

(16) $\frac{d}{dx}[\tan x]$

$$= \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{[\cos x]^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

(17)

a)

$$\begin{array}{ccccc} + & + & + & - & - & - \\ \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 7 & -6 & 4 & 7 & -6 \\ 3 & 5 & 5 & 3 & 5 \end{array}$$

$18\vec{k}$ $\quad -20\vec{i}$ $\quad -35\vec{j}$

$-30\vec{i}$ $\quad 12\vec{j}$ $\quad 35\vec{k}$

$$\boxed{\vec{A}\times\vec{B} = -50\vec{i} - 23\vec{j} + 53\vec{k}}$$

b)

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = 7(3) + (-6)(5) + 4(5)$
$= 11$

$A = \sqrt{7^2 + (-6)^2 + 4^2}$
$= \sqrt{101}$

$B = \sqrt{3^2 + 5^2 + 5^2}$
$= \sqrt{59}$

$11 = \sqrt{101}\sqrt{59}\cos\theta$

$$\cos\theta = \frac{11}{\sqrt{101}\sqrt{59}}$$

$$\theta = \cos^{-1}\left(\frac{11}{\sqrt{101}\sqrt{59}}\right)$$

$$\boxed{\theta \approx 81.8^\circ}$$

c)

$$\mathrm{proj}_{\vec{A}}(\vec{B}) = \frac{\vec{A}\cdot\vec{B}}{A^2}\vec{A}$$

$$= \frac{11}{101}(7\vec{i} - 6\vec{j} + 4\vec{k})$$

or $\frac{77}{101}\vec{i} - \frac{66}{101}\vec{j} + \frac{44}{101}\vec{k}$

18 a)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 3 & -4 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -1 & 2 & -2 & 1 & 0 \\ 0 & -6 & 10 & -3 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ R_2/(-1) \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -2 & 2 & -1 & 0 \\ 0 & -6 & 10 & -3 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_1-2R_2 \\ \\ R_3+6R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & -2 & 2 & -1 & 0 \\ 0 & 0 & -2 & 9 & -6 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ \\ R_3/(-2) \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & -2 & 2 & -1 & 0 \\ 0 & 0 & 1 & \frac{-9}{2} & 3 & \frac{-1}{2} \end{array}\right]$$

$$\begin{array}{l} R_1-R_3 \\ R_2+2R_3 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{2} & -1 & \frac{1}{2} \\ 0 & 1 & 0 & -7 & 5 & -1 \\ 0 & 0 & 1 & \frac{-9}{2} & 3 & \frac{-1}{2} \end{array}\right] \quad \begin{array}{l} -3+\frac{9}{2}=\frac{3}{2} \\ 2+2\left(\frac{-9}{2}\right)=-7 \\ \\ \end{array}$$

$= A^{-1}$

b) $AX=B$

$\boxed{X=A^{-1}B}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -1 & \frac{1}{2} \\ -7 & 5 & -1 \\ -\frac{9}{2} & 3 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -11 \\ -14 \\ 9 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$\boxed{\begin{array}{c} x=2 \\ y=-2 \\ z=3 \end{array}}$

or $\boxed{(x,y,z)=(2,-2,3)}$

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$$\left[\begin{array}{ccc|c} 2 & 8 & -10 & -2 \\ 3 & 5 & 6 & 4 \\ 4 & 2 & 22 & 10 \end{array}\right]$$

$$R_1/2 \quad \left[\begin{array}{ccc|c} 1 & 4 & -5 & -1 \\ 3 & 5 & 6 & 4 \\ 4 & 2 & 22 & 10 \end{array}\right]$$

$$\begin{array}{l} \\ R_2-3R_1 \\ R_3-4R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 4 & -5 & -1 \\ 0 & -7 & 21 & 7 \\ 0 & -14 & 42 & 14 \end{array}\right]$$

$$R_2/(-7) \quad \left[\begin{array}{ccc|c} 1 & 4 & -5 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & -14 & 42 & 14 \end{array}\right]$$

$$\begin{array}{l} \\ R_1-4R_2 \\ \\ R_3+14R_2 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & \# \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 7 & 3 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

RREF ✓

z = any real #
(no leading 1 in its column)

$x \quad +7z = 3$

$\boxed{x = 3-7z}$

$y - 3z = -1$

$\boxed{y = -1+3z}$

$$\boxed{\begin{array}{l} x = 3-7z \\ y = -1+3z \\ z = \text{any real \#} \end{array}}$$

or $\boxed{(x,y,z) = (3-7k, -1+3k, k)}$