Math 185 PRACTICE Quiz Four Total: 30 marks

Name:________________________________

1. [5 marks] Let $f(x) = (x^3 + x + 1)\tan^{-1}(2x - 1) + \pi\sin^{-1}x$. Find $f'(0)$.

$$f'(x) = (x^3+x+1)\cdot \frac{1}{1+(2x-1)^2}(2) + \tan^{-1}(2x-1)\cdot(3x^2+1) + \pi\cdot\frac{1}{\sqrt{1-x^2}}$$

$$f'(0) = 1\cdot\frac{1}{2}(2) + \tan^{-1}(-1)(1) + \pi\cdot 1$$

$$= 1 - \frac{\pi}{4}(1) + \pi$$

$$= 1 + \frac{3\pi}{4}$$

or $\frac{4+3\pi}{4}$

2. [4 marks] a) Use differentials to estimate the change in $y = \sin x$ as $x$ changes from $\frac{\pi}{3}$ to $\frac{17\pi}{48}$. Round your answer to four decimal places.

$dy = \cos x \, dx$ $\quad x = \frac{\pi}{3}$ $\quad dx = \frac{17\pi}{48} - \frac{\pi}{3} = \frac{\pi}{48}$

$$= \cos\frac{\pi}{3}\left(\frac{\pi}{48}\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{48}\right)$$

$$= \frac{\pi}{96} \simeq 0.0327$$

b) Use your calculator to find the true change, $\Delta y$, to four decimal places.

$$\Delta y = \sin\left(\frac{17\pi}{48}\right) - \sin\left(\frac{\pi}{3}\right)$$

$$\approx 0.0308$$

must be in radian mode

Quite a good approximation since $dy$ is very close to $\Delta y$

3. [4 marks] Find $f'(x)$ for $f(x) = \log_3[(x^7 + 5x^6)^8(x^2 + 9)]$. Simplify your answer.

SIMPLIFY FIRST

$$f(x) = \log_3 (x^7+5x^6)^8 + \log_3 (x^2+9)$$

$$= 8\log_3 (x^7+5x^6) + \log_3 (x^2+9)$$

$$f'(x) = 8 \cdot \frac{1}{\ln 3} \frac{1}{x^7+5x^6} (7x^6+30x^5) + \frac{1}{\ln 3} \cdot \frac{1}{x^2+9} (2x)$$

$$= \frac{8(7x^6+30x^5)(x^2+9) + 2x(x^7+5x^6)}{\ln 3 \, (x^7+5x^6)(x^2+9)}$$

$$= \frac{(56x^6+240x^5)(x^2+9) + 2x^8+10x^7}{\ln 3 \, (x^7+5x^6)(x^2+9)}$$

$$= \frac{58x^8+250x^7+504x^6+2160x^5}{\ln 3 \, (x^7+5x^6)(x^2+9)}$$

$$\underline{\text{or}} \quad \frac{2x^5(29x^3+125x^2+252x+1080)}{\ln 3 \, (x^7+5x^6)(x^2+9)}$$

4. [4 marks] Find $f'(y)$ for $\frac{e^{4y^2} - e^{-4y^2}}{e^{2y}}$. Simplify your answer.

SIMPLIFY FIRST

$$f(y) = e^{-2y}\left(e^{4y^2} - e^{-4y^2}\right)$$

$$f(y) = e^{4y^2-2y} - e^{-4y^2-2y}$$

$$f'(y) = e^{4y^2-2y}(8y-2) - e^{-4y^2-2y}(-8y-2)$$

$$\text{or} \quad (8y-2)e^{4y^2-2y} + (8y+2)e^{-4y^2-2y}$$

5. [10 marks] Integrate:

a) $\int (v^{11} + \frac{1}{v^{10}} + \frac{1}{\sqrt[3]{v}}) dv$

$$= \int (v^{11} + v^{-10} + v^{-1/3}) dv$$

$$= \frac{1}{12} v^{12} - \frac{1}{9} v^{-9} + \frac{3}{2} v^{2/3} + C$$

b) $\int (\frac{5\sqrt{x} - 6x^2}{x^6}) dx$

$$= \int x^{-6} (5x^{1/2} - 6x^2) dx$$

$$= \int (5x^{-11/2} - 6x^{-4}) dx$$

$$= 5\left(\frac{-2}{9} x^{-9/2}\right) - 6\left(\frac{-1}{3} x^{-3}\right) + C$$

$$= -\frac{10}{9} x^{-9/2} + 2x^{-3} + C$$

c) $\int t(t-2)^{98} dt$

Let $u = t - 2$

$du = dt$

$t = u + 2$

$$= \int (u+2) u^{98} du$$

$$= \int (u^{99} + 2u^{98}) du$$

$$= \frac{1}{100} u^{100} + \frac{2}{99} u^{99} + C$$

$$= \frac{1}{100} (t-2)^{100} + \frac{2}{99} (t-2)^{99} + C$$

6. [3 marks] Find $f(x)$ if $f''(x) = x^3 + 5x + 1$, $f(0) = 6$ and $f(1) = -1$.

$$f'(x) = \int (x^3 + 5x + 1)\,dx$$

$$f'(x) = \frac{1}{4}x^4 + \frac{5}{2}x^2 + x + C$$

$$f(x) = \int \left(\frac{1}{4}x^4 + \frac{5}{2}x^2 + x + C\right)dx$$

$$f(x) = \frac{1}{20}x^5 + \frac{5}{6}x^3 + \frac{1}{2}x^2 + Cx + D$$

different names for the 2 constants

Sub $x=0$
$f(x)=6$

$$6 = 0 + D$$
$$D = 6$$

$$f(x) = \frac{1}{20}x^5 + \frac{5}{6}x^3 + \frac{1}{2}x^2 + Cx + 6$$

Sub $x=1$
$f(x)=-1$

$$-1 = \frac{1}{20} + \frac{5}{6} + \frac{1}{2} + C + 6$$

$$-60 = \left(\frac{1}{20} + \frac{5}{6} + \frac{1}{2} + C + 6\right)60$$

$$-60 = 3 + 50 + 30 + 60C + 360$$

$$-503 = 60C$$

$$C = \frac{-503}{60}$$

$$\boxed{f(x) = \frac{1}{20}x^5 + \frac{5}{6}x^3 + \frac{1}{2}x^2 - \frac{503}{60}x + 6}$$