Math 185 PRACTICE Quiz Three Total: 30 marks

Name:________________________________

1. [4 marks] Approximate $\sqrt[3]{26.9}$ using a linear approximation.

$f(x) \approx f(a) + f'(a)\cdot(x-a)$

$f(x) = x^{1/3}$

$\boxed{a = 27}$

$f(27) = 3$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(27) = \frac{1}{3}\cdot\frac{1}{3^2}$

$= \frac{1}{27}$

$\sqrt[3]{26.9} \approx 3 + \frac{1}{27}(26.9 - 27)$

$\sqrt[3]{26.9} \approx 3 + \frac{1}{27}(-0.1)$

$\sqrt[3]{26.9} \approx 3 - \frac{1}{270}$

$\sqrt[3]{26.9} \approx \frac{809}{270}$ ← leave your answer exact

2. [3 marks] The radius r of a holograph is directly proportional to the square root of the wavelength λ of the light used. Show that $\frac{dr}{r} = \frac{1}{2}\frac{d\lambda}{\lambda}$.

$r = k\sqrt{\lambda}$ k constant

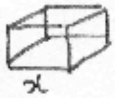
$r = k\lambda^{1/2}$

$\frac{dr}{r} = \frac{\frac{1}{2}k\lambda^{-1/2}\,d\lambda}{k\lambda^{1/2}}$

$\frac{dr}{r} = \frac{1}{2}\lambda^{-1}d\lambda$

$\frac{dr}{r} = \frac{1}{2}\frac{d\lambda}{\lambda}$ ✓

3. [4 marks] A metal cube dissolves in acid such that the length of each edge is decreasing by 0.25mm/min. How fast is the volume of the cube changing when the surface area of the cube is 24mm^2? Include units in your answer.



$\frac{dx}{dt} = -0.25$? $\frac{dV}{dt}$ $SA = 24$

x is <u>decreasing</u>

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$6x^2 = 24$

$x^2 = 4$

$x = \pm 2$

$x = 2$

$\frac{dV}{dt} = 3(2)^2(-0.25)$

$\frac{dV}{dt} = -3 \ \text{mm}^3/\text{min}$

Volume is decreasing

4. [6 marks] Find $\frac{dy}{dx}$ for the following functions:

a) $y = \sin(\cos(2\sqrt{x}))$

$$\frac{dy}{dx} = \cos(\cos(2\sqrt{x})) \cdot \frac{d}{dx}[\cos(2\sqrt{x})]$$

$$= \cos(\cos(2\sqrt{x})) \cdot -\sin(2\sqrt{x}) \cdot \frac{1}{\sqrt{x}}$$

$$= \frac{-\sin(2\sqrt{x})\cos(\cos(2\sqrt{x}))}{\sqrt{x}}$$

b) $y = \sqrt{\sec(3x) + \tan(4x)}$

$$y = [\sec(3x) + \tan(4x)]^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}[\sec(3x) + \tan(4x)]^{-1/2} \cdot \frac{d}{dx}[\sec(3x) + \tan(4x)]$$

$$= \frac{1}{2\sqrt{\sec(3x) + \tan(4x)}} \cdot [\sec(3x)\tan(3x)(3) + \sec^2(4x)4]$$

$$= \frac{3\sec(3x)\tan(3x) + 4\sec^2(4x)}{2\sqrt{\sec(3x) + \tan(4x)}}$$

5. [5 marks] The electric potential V on the line $3x + 2y = 6$ is given by $V = 3x^2 + 2y^2$. At what point on this line is the potential a minimum? Give exact values.

Minimize $V = 3x^2 + 2y^2$

$3x + 2y = 6$

$2y = 6 - 3x$

$y = 3 - 1.5x$

$\boxed{f(x) = 3x^2 + 2(3 - 1.5x)^2}$

$f'(x) = 6x + 4(3-1.5x)(-1.5)$

$= 6x - 6(3 - 1.5x)$

$= 6x - 18 + 9x$

$= 15x - 18$

Set $f'(x) = 0$

$15x - 18 = 0$

$3(5x - 6) = 0$

$x = \frac{6}{5}$

Check: $f''(x) = 15 > 0$

min ✓

$x = \frac{6}{5}$

$y = 3 - 1.5\left(\frac{6}{5}\right)$

$= 3 - \frac{9}{5}$

$= \frac{6}{5}$

$\boxed{\text{The point is } \left(\frac{6}{5}, \frac{6}{5}\right).}$

6. [8 marks] Let $f(x) = x^5 - 5x + 12$.

a) Find $f'(x)$

$$f'(x) = 5x^4 - 5$$

b) Find $f''(x)$

$$f''(x) = 20x^3$$

c) Find all relative maximum and relative minimum points. Indicate whether each point is a maximum or a minimum.

Set $f'(x) = 0$

$$5x^4 - 5 = 0$$

$$5(x^4 - 1) = 0$$

$$5(x^2-1)(x^2+1) = 0$$

$$5(x-1)(x+1)(x^2+1) = 0 \quad x = \pm 1$$

-1 1

$f'(x) > 0$ $f'(x) < 0$ $f'(x) > 0$

Rel. Max @ $(-1, 16)$

Rel. Min @ $(1, 8)$

d) On which interval(s) is $f(x)$ decreasing?

$\boxed{f'(x) < 0}$

$-1 < x < 1$

or $(-1, 1)$

e) Find all points of inflection.

$$f''(x) = 0$$

$$20x^3 = 0$$

$$x = 0$$

$(0, 12)$

f) Sketch the curve with all points from parts c) and d) labelled.