Math 185 PRACTICE Quiz Two   Total: 30 marks

Name:________________________________

1. [[illegible] marks] Differentiate with respect to the given variable:

a) $f(x) = \frac{1}{x^6}$ $= x^{-6}$

$$f'(x) = -6x^{-7}$$

b) $g(v) = \sqrt[7]{v^4}$ $= v^{4/7}$

$$g'(v) = \frac{4}{7} v^{-3/7}$$

c) $h(t) = \frac{1}{(5t^3+6t^2+4)^2}$ $= (5t^3+6t^2+4)^{-2}$

$$h'(t) = -2(5t^3+6t^2+4)^{-3}(15t^2+12t) = \frac{-2(15t^2+12t)}{(5t^3+6t^2+4)^3}$$

d) Where is $h'(t) = 0$?

$$-2(15t^2+12t) = 0$$

$$-6t(5t+4) = 0 \qquad t=0,\ t=-\frac{4}{5}$$

2. [[illegible] marks] Find $\frac{dy}{dx}\big|_{x=-1}$ for $y = (4x^3+2)^8(7x^4+3x)$.

$$\frac{dy}{dx} = (4x^3+2)^8(28x^3+3) + (7x^4+3x)\frac{d}{dx}\left[(4x^3+2)^8\right]$$

$$\frac{dy}{dx} = (4x^3+2)^8(28x^3+3) + (7x^4+3x)\,8(4x^3+2)^7(12x^2)$$

$$\frac{dy}{dx}\bigg|_{x=-1} = (-2)^8(-25) + (4)\,8(-2)^7(12)$$

$$= -6400 - 49{,}152$$

$$= -55{,}552$$

3. [2 marks] We want to solve $x^6 + x - 32 = 0$. Use Newton's Method with $x_0 = -2$ [redacted] to find x_1. Round your answer to two decimal places.

$$f(x) = x^6 + x - 32$$
$$f'(x) = 6x^5 + 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= -2 - \frac{[(-2)^6 + (-2) - 32]}{[6(-2)^5 + 1]}$$

$$\approx -1.84$$

4. [4 marks] A particle's position in metres is given by $x = 6 + t + t^2, y = \frac{54}{t^2}$. Find the particle's velocity at $t = 3$ seconds. Include units in your answer.

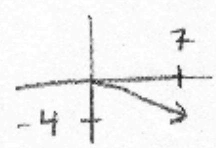
$$y = 54t^{-2}$$

$$V_x = 1 + 2t \qquad V_y = -108t^{-3}$$

$$@\ t = 3 \quad V_x = 7 \quad V_y = -4$$

$$V = \sqrt{(7)^2 + (-4)^2} \approx 8.1$$

$$\tan^{-1}\left(\frac{V_y}{V_x}\right) \approx -29.7°$$



Correct quadrant ✓

$$\boxed{\begin{array}{l} 8.1 \text{ m/s at } -29.7° \\ \text{or } 330.3° \end{array}}$$

5. [4 marks] Find $f'(x)$ for $f(x) = (3x^2)\sqrt{5x^7}$. Simplify your answer.

$$f(x) = (3x^2)(5x^7)^{1/2}$$

$$f'(x) = (3x^2)\underbrace{\tfrac{1}{2}(5x^7)^{-1/2}(35x^6)}_{\text{chain Rule}} + (5x^7)^{1/2}(6x)$$

$$= \frac{(3x^2)(35x^6) + 2(5x^7)(6x)}{2(5x^7)^{1/2}}$$

Common denominator

$$= \frac{105x^8 + 60x^8}{2(5x^7)^{1/2}}$$

$$= \frac{165x^8}{2(5x^7)^{1/2}}$$

6. [6 marks] Find the equation of:

a) the tangent line to $y = \frac{x+2}{x^2+2}$ at the point $(2, \frac{2}{3})$. Leave your answer in the form $ax + by + c = 0$.

$$y' = \frac{(x^2+2)(1) - (x+2)(2x)}{(x^2+2)^2}$$

$$y'|_{x=2} = \frac{(6)(1) - (4)(4)}{(6)^2}$$

$$= \frac{-10}{36} \text{ or } \frac{-5}{18}$$

$$y - \frac{2}{3} = \frac{-5}{18}(x-2)$$

$$18y - 12 = -5(x-2)$$

$$18y - 12 = -5x + 10$$

$$\boxed{5x + 18y - 22 = 0}$$

b) the normal line to $y = \frac{x+2}{x^2+2}$ at the point $(2, \frac{2}{3})$. Leave your answer in the form $ax + by + c = 0$.

$$m_{normal} = \frac{18}{5} \qquad m_{normal} = \frac{-1}{m_{tan}}$$

$$y - \frac{2}{3} = \frac{18}{5}(x-2)$$

$$15y - 10 = 18(3)(x-2)$$

$$15y - 10 = 54x - 108$$

$$\boxed{-54x + 15y + 98 = 0}$$

or $54x - 15y - 98 = 0$

7. [4 marks] Find $\frac{dy}{dx}$ for the curve $x^6 + 4y^2 + (2y)(x^3 - 1)^5 = 5x$.

$$6x^5 + 8y\frac{dy}{dx} + \underbrace{2y\left[5(x^3+1)^4(3x^2)\right] + (x^3+1)^5\left(2\frac{dy}{dx}\right)}_{\text{Product Rule}} = 5$$

$$6x^5 + 8y\frac{dy}{dx} + 30x^2y(x^3+1)^4 + 2(x^3+1)^5\frac{dy}{dx} = 5$$

$$\left[8y + 2(x^3+1)^5\right]\frac{dy}{dx} = 5 - 6x^5 - 30x^2y(x^3+1)^4$$

$$\frac{dy}{dx} = \frac{5 - 6x^5 - 30x^2y(x^3+1)^4}{8y + 2(x^3+1)^5}$$