Math 185 PRACTICE Quiz One   Total: 30 marks

Name:____________________________

1. [4 marks] State the values of x for which the functions below are continuous:

a) $f(x) = \frac{6}{2x-17}$ $x \neq \frac{17}{2}$

[This means all real values except $\frac{17}{2}$].

b) $f(x) = \begin{cases} x-1, & x \leq -3 \\ 3x^2, & -3 < x < -1 \\ 4+x, & x \geq -1 \end{cases}$ $x \neq -3$

As $x \to -3$ from the left $f(x) \to -4$

" right $f(x) \to 27$

As $x \to -1$ from the left $f(x) \to 3$

" right $f(x) \to 3$

$f(x)$ is continuous @ $x = -1$

2. [3 marks] Evaluate the following limit: $\lim_{x \to -1} \frac{x^2-1}{x^2-3x-4}$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x-4)}$$

$$= \lim_{x \to -1} \frac{(x-1)}{(x-4)}$$

$$= \frac{-2}{-5}$$

$$= \frac{2}{5}$$

3. [3 marks] Find $f'(2)$ for $f(x) = 5x^4 - 16x^3 + x^2$.

$$f'(x) = 20x^3 - 48x^2 + 2x$$

$$f'(2) = 20(2)^3 - 48(2)^2 + 2(2)$$

$$= -28$$

4. [4 marks] An object's displacement (in metres) is given by $s(t) = t^4 + 7t^3 + 1$, where t is measured in seconds. Find the acceleration of the object. Include correct units.

$v(t) = s'(t) = 4t^3 + 21t^2$ (m/s)

$a(t) = s''(t) = 12t^2 + 42t$ (m/s^2)

5. [4 marks] An object's displacement (in metres) is given by $s(t) = s_0 + v_0 t - 4.9t^2$, where t is measured in seconds; s_0 and v_0 are constants.

a) Find the object's velocity at $t = 3$ seconds.

$$v(t) = s'(t) = v_0 - 9.8t$$

$$v(3) = v_0 - 9.8(3)$$

$$= v_0 - 29.4 \quad m/s$$

b) Find the time at which the object's velocity is zero.

Set $v(t) = 0$ $\qquad v_0 - 9.8t = 0$

$$t = \frac{v_0}{9.8} \quad s$$

6. [8 marks] Differentiate with respect to the given variable:

a) $h(v) = \frac{3v^2+4}{2v+1}$

$$h'(v) = \frac{(2v+1)(6v) - (3v^2+4)(2)}{(2v+1)^2}$$

$$= \frac{12v^2+6v - (6v^2+8)}{(2v+1)^2}$$

$$= \frac{6v^2+6v-8}{(2v+1)^2}$$

or $\dfrac{2(3v^2+3v-4)}{(2v+1)^2}$

b) $u(r) = (r^3+1)(6r^2-2)$

$$u'(r) = (r^3+1)(12r) + (6r^2-2)(3r^2)$$

$$= (12r^4+12r) + (18r^4-6r^2)$$

$$= 30r^4 - 6r^2 + 12r$$

$$\text{or } 6r(5r^3 - r + 2)$$

7. [4 marks] Use the **definition** of the derivative to find $f'(x)$ for $f(x) = 5x^2 + \frac{2}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[5(x+h)^2 + \frac{2}{x+h} - 5x^2 - \frac{2}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[5(x^2+2xh+h^2) - 5x^2 + \frac{2}{x+h} - \frac{2}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[5x^2 + 10xh + 5h^2 - 5x^2 + \frac{2x - 2(x+h)}{(x+h)x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[10xh + 5h^2 \quad - \frac{2h}{(x+h)x}\right]$$

$$= \lim_{h \to 0} \quad 10x + 5h - \frac{2}{(x+h)x}$$

$$= 10x - \frac{2}{x^2}$$