13. $y = 2x^2 + 5x$ $m = 13$

$y' = 4x + 5$

Set $13 = 4x + 5$ to find x:

$8 = 4x$

$x = 2$

When $x = 2$ $y = 2(2)^2 + 5(2)$

$y = 18$ $(x_1, y_1) = (2, 18)$

$y - y_1 = m(x - x_1)$

$y - 18 = 13(x - 2)$

$y - 18 = 13x - 26$

$\boxed{y = 13x - 8}$

14. $y = (3x^2+4)^{-1}$

$y' = -(3x^2+4)^{-2}(6x)$

$y' = \dfrac{-6x}{(3x^2+4)^2}$ Given $(x_1, y_1) = (-1, \frac{1}{7})$

when $x = -1$ $m_{tan} = \dfrac{-6(-1)}{(3(-1)^2+4)^2}$

$= \dfrac{6}{7^2}$

$= \dfrac{6}{49}$

$m_{normal} = -\dfrac{49}{6}$

Tangent Line

$y - y_1 = m(x - x_1)$

$y - \frac{1}{7} = \frac{6}{49}(x+1)$

$49y - 7 = 6(x+1)$

$49y - 7 = 6x + 6$

$\boxed{6x - 49y + 13 = 0}$

Normal Line

$y - y_1 = m(x - x_1)$

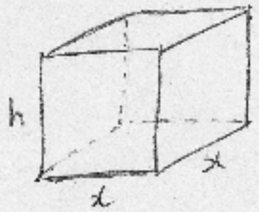
$y - \frac{1}{7} = \frac{-49}{6}(x+1)$

$42y - 6 = -343(x+1)$

$42y - 6 = -343x - 343$

$\boxed{343x + 42y + 337 = 0}$

15.



Let x = length of sides of the base

Let h = height

Given volume = 13,500 cm^3

$$x^2 h = 13,500$$

$$h = \frac{13,500}{x^2}$$

Maximize $SA = x^2 + 4xh$

$$f(x) = x^2 + \frac{54,000x}{x^2}$$

$$= x^2 + \frac{54,000}{x}$$

$$f'(x) = 2x - \frac{54,000}{x^2}$$

$$= \frac{2x^3 - 54,000}{x^2}$$

Set $f'(x) = 0$: $2x^3 - 54,000 = 0$

$$x^3 = 27,000$$

$$x = 30$$

Check: $f''(x) = 2 + \frac{108,000}{x^3}$

$f''(30) > 0$ MINIMUM

$h = \frac{13,500}{30^2} = 15$

$\boxed{\text{Dimensions are } 30cm \times 30cm \times 15cm}$

16. a) $f'(x) = 24x^2 - 144x - 648$

b) $f''(x) = 48x - 144$

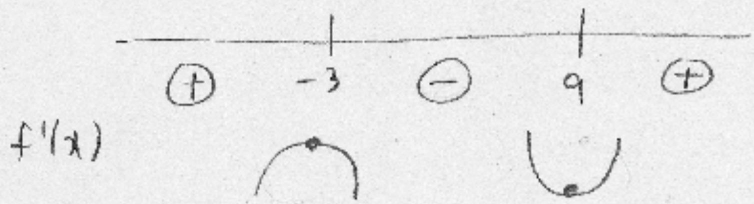
c) Set $f'(x) = 0$

$$24x^2 - 144x - 648 = 0$$

$$24(x^2 - 6x - 27) = 0$$

$$24(x-9)(x+3) = 0$$

$$x = 9, -3$$



Relative Maximum at $(-3, \underset{f(-3)}{\underset{\uparrow}{1080}})$

Relative Minimum at $(9, \underset{f(9)}{\underset{\uparrow}{-5832}})$

d) Set $f''(x)=0$

$$48x-144=0$$

$$x=3$$

Point of inflection $(3,-2376)$

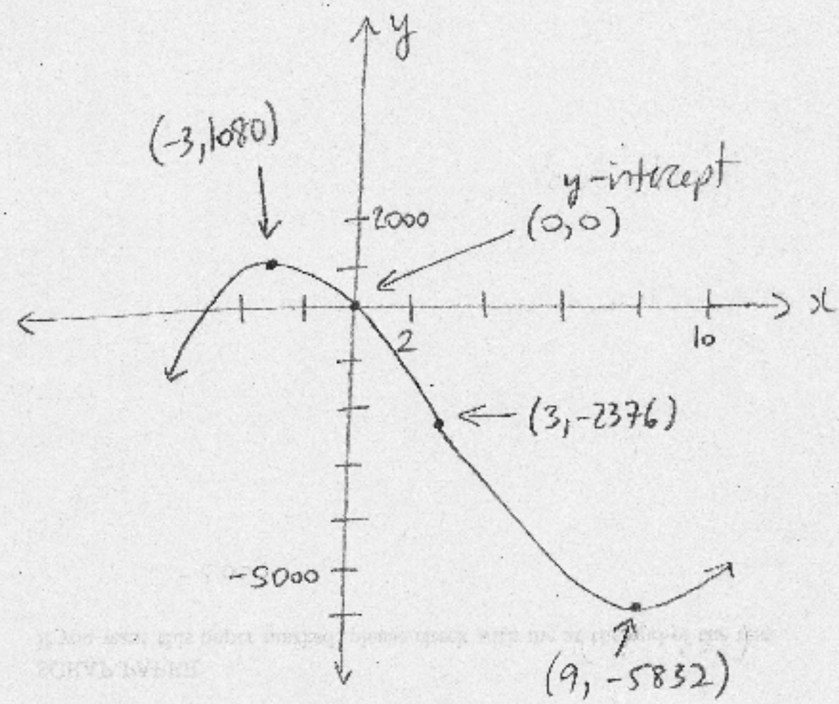
$\uparrow$ $f(3)$

e)

		3	
$f''(x)$	$\ominus$		$\oplus$
	$\cap$		$\cup$

Concave up on $(3,\infty)$ or $3<x<\infty$

f)



17. $V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 0.2$ cm/s

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

? $\frac{dV}{dt}$ when $V = 2304\pi$

$\frac{4}{3}\pi r^3 = 2304\pi$

$r^3 = 1728$

$r = 12$

$\frac{dV}{dt} = 4\pi(12)^2(0.2)$

$\boxed{\frac{dV}{dt} = 115.2\pi \text{ cm}^3/\text{s}}$

18. $f(x) = x^{1/4}$

$f'(x) = \frac{1}{4}x^{-3/4}$

$\boxed{f(x) \approx f(a) + f'(a)(x-a)}$

$x = 15.8$

Choose $a = 16$

$f(a) = 2$

$f'(a) = \frac{1}{4} \cdot \frac{1}{\sqrt[4]{16}^{\,3}}$

$= \frac{1}{4} \cdot \frac{1}{8}$

$= \frac{1}{32}$

$\sqrt[4]{15.8} \approx 2 + \frac{1}{32}(15.8 - 16)$

$= 2 + \frac{1}{32}(-0.2)$

$= 2 + \frac{1}{32}\left(\frac{-2}{10}\right)$

$= \frac{638}{320}$ or $\frac{319}{160}$

19. $A = \pi r^2$

? $\frac{dr}{r}$

Given $\frac{dA}{A} = 0.07$

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi r\, dr$$

$$\frac{dA}{A} = \frac{2\pi r\, dr}{\pi r^2}$$

$$\frac{dA}{A} = 2\frac{dr}{r}$$

$$0.07 = 2\frac{dr}{r}$$

$$\boxed{0.035 = \frac{dr}{r}}$$

20. $y = \cos^3(7x-2) + \sec(5\sqrt{x}+1)$

$$y = [\cos(7x-2)]^3 + \sec(5\sqrt{x}+1)$$

$$\frac{dy}{dx} = 3[\cos(7x-2)]^2 \cdot [-\sin(7x-2)\cdot 7]$$

$$+ \sec(5\sqrt{x}+1)\tan(5\sqrt{x}+1)\left[\frac{5}{2}x^{-1/2}\right]$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 3(\cos 5)^2(-7\sin 5) + \sec 6 \tan 6 \left(\frac{5}{2}\right)$$

$$= -21(\cos 5)^2 \sin 5 + \frac{5}{2}\,\frac{\tan 6}{\cos 6}$$

$$\sec 6 = \frac{1}{\cos 6}$$

Make sure your calculator is in radian mode

$$\left.\frac{dy}{dx}\right|_{x=1} \approx 0.86$$

21. $f(x) = (2x+5)\cos^{-1}x + \frac{1}{3}\tan^{-1}(8x+1)$

$$f'(x) = (2x+5)\cdot\frac{-1}{\sqrt{1-x^2}} + \cos^{-1}x\cdot(2) + \frac{1}{3}\cdot\frac{1}{1+(8x+1)^2}\cdot 8$$

$$f'(0) = -5 + 2\cos^{-1}(0) + \frac{8}{3}\cdot\frac{1}{1+1}$$

$$= -5 + 2\left(\frac{\pi}{2}\right) + \frac{4}{3}$$

$$= \frac{-11}{3} + \pi$$

$\cos^{-1}0 = \frac{\pi}{2}$

or $\dfrac{-11+3\pi}{3}$

22. $0 \le \theta \le \frac{\pi}{2}$

Maximize $f(\theta) = 4\sin\theta + 7\cos\theta$

$$f'(\theta) = 4\cos\theta - 7\sin\theta$$

Set $f'(\theta) = 0$:

$$0 = 4\cos\theta - 7\sin\theta$$

$$7\sin\theta = 4\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{4}{7}$$

$$\tan\theta = \frac{4}{7}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{4}{7}\right)}$$

Check: $f''(\theta) = -4\sin\theta - 7\cos\theta$

$\theta \approx 29.7°$ $\quad f''(29.7°) < 0$ $\quad$ MAXIMUM ✓

23. $f(x) = 5^{4x^2-3x}$

$$f'(x) = \ln 5 \cdot 5^{4x^2-3x} \cdot \frac{d}{dx}[4x^2-3x]$$

$$= \ln 5 \cdot 5^{4x^2-3x} \cdot (8x-3)$$

24. $f(x) = \ln \frac{\sqrt{x^2+3}}{\sqrt{5x-4}}$

$$= \ln\sqrt{x^2+3} - \ln\sqrt{5x-4}$$

$$= \frac{1}{2}\ln(x^2+3) - \frac{1}{2}\ln(5x-4)$$

$$f'(x) = \frac{1}{2} \cdot \frac{2x}{x^2+3} - \frac{1}{2} \cdot \frac{5}{5x-4}$$

$$= \frac{x}{x^2+3} - \frac{5}{10x-8}$$

$$= \frac{x(10x-8) - 5(x^2+3)}{(x^2+3)(10x-8)}$$

$$= \frac{10x^2-8x-5x^2-15}{(x^2+3)(10x-8)}$$

$$= \frac{5x^2-8x-15}{(x^2+3)(10x-8)}$$

or $\frac{5x^2-8x-15}{2(x^2+3)(5x-4)}$