1. a) $\displaystyle \lim_{x \to -6} \frac{x^2+2x-24}{x^2+11x+30}$

$\displaystyle = \lim_{x \to -6} \frac{(x+6)(x-4)}{(x+6)(x+5)}$

$\displaystyle = \lim_{x \to -6} \frac{x-4}{x+5}$

$\displaystyle = \frac{-10}{-1}$

$= 10$

b) $\displaystyle \lim_{x \to \infty} \frac{3x^2-2}{1+x^2}$

$\displaystyle = \lim_{x \to \infty} \frac{\frac{3x^2}{x^2} - \frac{2}{x^2}}{\frac{1}{x^2} + \frac{x^2}{x^2}}$

$\displaystyle = \lim_{x \to \infty} \frac{3 - \frac{2}{x^2}}{\frac{1}{x^2} + 1}$

$\displaystyle = \frac{3-0}{0+1}$

$= 3$

2. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h\to 0} \frac{1}{h}\left[2(x+h) + \frac{3}{x+h-1} - \left(2x + \frac{3}{x-1}\right)\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[2x + 2h + \frac{3}{x+h-1} - 2x - \frac{3}{x-1}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[2h + \frac{3}{x+h-1} - \frac{3}{x-1}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[2h + \frac{3(x-1) - 3(x+h-1)}{(x+h-1)(x-1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[2h + \frac{3x-3-3x-3h+3}{(x+h-1)(x-1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[2h - \frac{3h}{(x+h-1)(x-1)}\right]$$

$$= \lim_{h\to 0} \left[2 - \frac{3}{(x+h-1)(x-1)}\right]$$

$$= 2 - \frac{3}{(x-1)^2}$$

3. $f'(x) = 6x^5 - 9x^2 + 10x$

$f'(-2) = 6(-2)^5 - 9(-2)^2 + 10(-2)$

$= 6(-32) - 9(4) - 20$

$= -248$

4. $s(t) = (t^3 + 2t + 2)(5t^2 + 6)$

$s'(t) = (t^3 + 2t + 2)(10t) + (5t^2 + 6)(3t^2 + 2)$

by Product Rule

$= 10t^4 + 20t^2 + 20t + (15t^4 + 10t^2 + 18t^2 + 12)$

$= 25t^4 + 48t^2 + 20t + 12$ m/s

or $v(t) = 25t^4 + 48t^2 + 20t + 12$ m/s

5.

$$f'(x) = a(8x^3 + 21bx^2 + c)$$

$$f''(x) = a(24x^2 + 42bx)$$

Set $f''(x) = 0$

$$a(24x^2 + 42bx) = 0$$

$$6ax(4x + 7b) = 0$$

$$\swarrow \qquad \downarrow$$

$$6ax = 0 \qquad 4x + 7b = 0$$

$$x = 0 \qquad 4x = -7b$$

$$x = -\frac{7b}{4}$$

$$x = 0, \ -\frac{7b}{4}$$

6. $f(x) = x^5(x+3)\,(x^3+2)$

$f(x) = (x^6 + 3x^5)(x^3+2)$

$f'(x) = (x^6+3x^5)(3x^2) + (x^3+2)(6x^5+15x^4)$

by Product Rule

$= (3x^8 + 9x^7) + (6x^8 + 15x^7 + 12x^5 + 30x^4)$

$= 9x^8 + 24x^7 + 12x^5 + 30x^4$

7. $$y = \frac{x^2-21}{x+5}$$

$$\frac{dy}{dx} = \frac{(x+5)(2x) - (x^2-21)(1)}{(x+5)^2}$$ by Quotient Rule

$$= \frac{2x^2+10x - x^2+21}{(x+5)^2}$$

$$= \frac{x^2+10x+21}{(x+5)^2}$$

b) set $\frac{dy}{dx} = 0$

$$\frac{x^2+10x+21}{(x+5)^2} = 0$$

$$\frac{(x+3)(x+7)}{(x+5)^2} = 0$$

$$(x+3)(x+7) = 0$$

$$x = -3, -7$$

c) When $\frac{dy}{dx}$ is defined

For all real numbers $x \neq -5$

8. $f'(x) = -2\left[(2x+1)^4+6x\right]^{-3} \cdot \frac{d}{dx}\left[(2x+1)^4+6x\right]$

$= -2\left[(2x+1)^4+6x\right]^{-3} \cdot \left[4(2x+1)^3(2)+6\right]$

$f'(-1) = -2\left[(-1)^4 \quad -6\right]^{-3}\left[4(-1)^3(2)+6\right]$

$= -2\left[-5\right]^{-3}(-8+6)$

$= \frac{4}{(-5)^3}$

$= \frac{4}{-125}$

$= \frac{-4}{125}$

9. $x^3 - 5\sqrt{x} - 50 = 0$

$$f(x) = x^3 - 5x^{1/2} - 50 \qquad f'(x) = 3x^2 - \frac{5}{2}x^{-1/2}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad x_0 = 4$$

$$= 4 - \frac{f(4)}{f'(4)}$$

$$= 4 - \frac{[4^3 - 5\sqrt{4} - 50]}{[3(4)^2 - \frac{5}{2} \cdot \frac{1}{\sqrt{4}}]}$$

$$\approx 3.91$$

10. $x^4 + y^3 - 6(x^2+3)y = 11x$

$x^4 + y^3 + [-6(x^2+3)]y = 11x$

Take $\frac{d}{dx}$ of both sides:

$$4x^3 + 3y^2 \cdot \frac{dy}{dx} + [-6(x^2+3)]\frac{dy}{dx} + y[-6(2x)] = 11$$

$$3y^2 \frac{dy}{dx} - 6(x^2+3)\frac{dy}{dx} = 11 - 4x^3 + 12xy$$

$$[3y^2 - 6x^2 - 18]\frac{dy}{dx} = 11 - 4x^3 + 12xy$$

$$\frac{dy}{dx} = \frac{11 - 4x^3 + 12xy}{3y^2 - 6x^2 - 18}$$

11.

$$f(x) = (7x^4+1)^{1/2}(5-3x)$$

$$f'(x) = (7x^4+1)^{1/2}(-3) + (5-3x)\cdot\tfrac{1}{2}(7x^4+1)^{-1/2}(28x^3)$$

$$= -3\sqrt{7x^4+1} + \frac{(5-3x)(28x^3)}{2\sqrt{7x^4+1}}$$

$$= \frac{-3(7x^4+1)}{\sqrt{7x^4+1}} + \frac{(5-3x)(14x^3)}{\sqrt{7x^4+1}}$$

$$= \frac{-21x^4-3+70x^3-42x^4}{\sqrt{7x^4+1}}$$

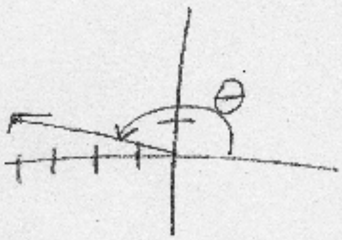
$$= \frac{-63x^4+70x^3-3}{\sqrt{7x^4+1}}$$

12. $x = t^3 - 4t^2 \qquad y = 1 + \frac{t^2}{4}$

$v_x = 3t^2 - 8t \qquad v_y = \frac{t}{2}$

@ $t = 2$:

$v_x = -4 \qquad v_y = 1$



$v = \sqrt{(-4)^2 + 1^2}$

≈ 4.1 m/s

$\tan^{-1}\left(\frac{v_y}{v_x}\right) \approx -14.0°$

From diagram $\theta \approx -14.0° + 180°$

$\approx 166.0°$

$\boxed{v = 4.1 \text{ m/s at } 166.0°}$