

2011 Test Questions for Math 185

1. Evaluate the following limits:

a)  $\lim_{x \rightarrow -6} \frac{x^2 + 2x - 24}{x^2 + 11x + 30}$

b)  $\lim_{x \rightarrow \infty} \frac{3x^2 - 2}{1 + x^2}$

2. Let  $f(x) = 2x + \frac{3}{x-1}$ . Find  $f'(x)$  using the limit definition.

3. Find the slope of the tangent line to  $f(x) = x^6 - 3x^3 + 5x^2$  at the point  $x = -2$ .

4. An object's displacement (in metres) is given by  $s = (t^3 + 2t + 2)(5t^2 + 6)$ , where  $t$  is measured in seconds. Find the instantaneous rate of change of the object's displacement. Include the correct units, and simplify your answer.

5. Let  $f(x) = a(2x^4 + 7bx^3 + cx)$ , where  $a, b$  and  $c$  are all nonzero constants. Find the second derivative of  $f(x)$ . For which  $x$ -values is  $f''(x) = 0$ ?

6. Differentiate  $f(x) = x^5(x + 3)(x^3 + 2)$ . Simplify your answer.

7. [6 marks] a) Find  $\frac{dy}{dx}$  for  $y = \frac{x^2 - 21}{x + 5}$

b) For which  $x$ -values is the tangent line to  $y$  horizontal?

c) For which  $x$ -values is  $y$  differentiable?

8. Find  $f'(-1)$  for  $f(x) = [(2x + 1)^4 + 6x]^{-2}$

9. We want to solve  $x^3 - 5\sqrt{x} = 50$  using Newton's Method. Find  $x_1$  using  $x_0 = 4$ . Round your answer to 2 decimal places.

10. Find  $\frac{dy}{dx}$  for  $x^4 + y^3 - 6(x^2 + 3)y = 11x$

11. Find  $f'(x)$ . Simplify your answer as much as possible.

$$f(x) = \sqrt{7x^4 + 1}(5 - 3x)$$

12. An object's position (in metres) at  $t$  seconds is given by  $x = t^3 - 4t^2, y = 1 + \frac{1}{4}t^2$ . Find the object's velocity at  $t = 2$  seconds.

13. Find the equation of the tangent line to  $y = 2x^2 + 5x$  at the point where the slope of the tangent line is 13. Write your answer in the form  $y = mx + b$ .

14. Find the equations of the tangent line and normal line to the curve  $y = \frac{1}{3x^2+4}$  at the point  $(-1, \frac{1}{7})$ . Write the lines in the form  $ax + by + c = 0$ .

15. A rectangular box is open at the top and has a square base. The volume of the box is  $13,500 \text{ cm}^3$ . Find the dimensions of the box that minimize its surface area.

16. Let  $f(x) = 8x^3 - 72x^2 - 648x$ .

a) Find  $f'(x)$

b) Find  $f''(x)$

c) Find all relative maximum and relative minimum points. Indicate whether each point is a maximum or a minimum.

d) Find all points of inflection.

e) On which interval(s) is  $f(x)$  concave up?

f) Sketch the curve with all points from parts c) and d) labelled.

17. A spherical balloon is being inflated so that the radius is increasing at a rate of  $0.2 \text{ cm}$  per second. At what rate is the volume changing at the moment when the volume is  $2304\pi \text{ cm}^3$ ? Include units in your answer.

18. Approximate  $\sqrt[4]{15.8}$  using a linear approximation. Express your final answer as a fraction.

19. Let  $A$  be the area of a circle with radius  $r$ . Find  $\frac{dr}{r}$  given that  $\frac{dA}{A} = 0.07$ .

20. Find  $\frac{dy}{dx}|_{x=1}$  for the following function, rounding your answer to 2 decimal places:  $y = \cos^3(7x - 2) + \sec(5\sqrt{x} + 1)$

21. Find  $f'(0)$  for  $f(x) = (2x + 5) \cos^{-1} x + \frac{1}{3} \tan^{-1}(8x + 1)$

22. Find the angle  $\theta$  between  $0$  and  $\frac{\pi}{2}$  at which the function  $f(\theta) = 4 \sin \theta + 7 \cos \theta$  is maximized. Leave your answer in exact form.

23. Find  $f'(x)$  for  $f(x) = 5^{4x^2-3x}$ .

24. Find  $f'(x)$  for  $f(x) = \ln \sqrt{\frac{x^2+3}{5x-4}}$ . Simplify your answer.

25. (Extra Practice) Let  $f(x) = \frac{x}{4-x^2}$

- a) Find all intercepts
- b) Find all asymptotes
- c) Find  $f'(x)$
- d) Find all relative maximum and minimum points (if there are any)
- e) On what intervals is  $f(x)$  increasing?
- f) Sketch the graph

26. (Extra Practice) Find  $\frac{dy}{dx}$  for  $\sin(xy) + \cos(2y) = x^2$

27. (Extra Practice) Find  $y'$  for  $y = \frac{3(e^{2x} - e^{-2x})}{e^{2x}}$