2015

Math 172–Quiz # 5

Name:_____ **Total: 40 Points**

1. [2 points] State the domain of the following rational expression in either set-builder notation or interval notation.

$\frac{x^2-9}{x^2+7x}$

$\{x \mid x \neq 0 \text{ and } x \neq -7\}$

$x^2+7x=0$

$x(x+7)=0$

$x=0$ $\quad x=-7$

or $(-\infty,-7)\cup(-7,0)\cup(0,\infty)$

2. [3 points] Reduce the expression to lowest terms.

$\frac{3a-15b}{25ab^2-a^3}$

$= \frac{3(a-5b)}{a(25b^2-a^2)}$

$= \frac{3(a-5b)^{-1}}{a(5b-a)(5b+a)}$

$= \frac{-3}{a(5b+a)}$

$\frac{-3}{a(5b+a)}$

3. [8 points] Perform the indicated operations. Express your answer in lowest terms.

a) $\frac{2x^2+5x-3}{6x-3} \cdot \frac{x^2-9}{6}$

← ac method: 6, −1

$2x^2+6x-x-3$

$= 2x(x+3)-1(x+3)$

$= (2x-1)(x+3)$

$= \frac{(2x-1)(x+3)}{3(2x-1)} \times \frac{6}{(x-3)(x+3)}^{2}$

$= \frac{2}{x-3}$

$\frac{2}{x-3}$

(4)

$\frac{28}{(z-4)(z+4)(z+3)}$

(4)

b) $\frac{4}{z^2-16} - \frac{4}{z^2+7z+12}$

$= \frac{4}{(z-4)(z+4)} - \frac{4}{(z+3)(z+4)}$

$= \frac{4(z+3)}{(z-4)(z+4)(z+3)} - \frac{4(z-4)}{(z+3)(z+4)(z-4)}$

$= \frac{4z+12-4z+16}{(z-4)(z+4)(z+3)} \longrightarrow = \frac{28}{(z-4)(z+4)(z+3)}$

4. [4 points] Simplify the following complex fraction:

$\frac{\frac{5}{a-1} - \frac{2}{a+1}}{\frac{1}{1-a} + \frac{6}{a+1}}$

$\frac{3a+7}{5a-7}$

$= \frac{\left(\frac{5}{a-1} - \frac{2}{a+1}\right)}{\left(\frac{-1}{a-1} + \frac{6}{a+1}\right)} \cdot \frac{(a-1)(a+1)}{(a-1)(a+1)}$

$= \frac{5(a+1) - 2(a-1)}{-1(a+1) + 6(a-1)}$

$= \frac{5a+5-2a+2}{-a-1+6a-6}$

$= \frac{3a+7}{5a-7}$

5. [3 points] Solve the following equation for q.

$\frac{1}{p} + \frac{2}{q} = \frac{3}{r}$

$\frac{-2pr}{r-3p}$ or $\frac{2pr}{3p-r}$

$pqr\left(\frac{1}{p} + \frac{2}{q}\right) = \frac{3}{r} \cdot pqr$

$qr + 2pr = 3pq$

$qr - 3pq = -2pr$

$q(r-3p) = -2pr$

$q = \frac{-2pr}{r-3p}$

or $q = \frac{2pr}{3p-r}$

6. [9 points] Find the solution set for the following equations:

a) $\frac{y+2}{3} = \frac{4}{y-2}$

$\{4, -4\}$

$(y+2)(y-2) = 12$

$y^2 - 4 = 12$

$y^2 - 16 = 0$

$(y-4)(y+4) = 0$

$y-4=0$ or $y=-4$

$y=4$

Check $y=4$: $LS = \frac{6}{3} = 2$ ✓

$RS = \frac{4}{2} = 2$

Check $y=-4$: $LS = \frac{-2}{3}$

$RS = \frac{4}{-6} = \frac{-2}{3}$ ✓

4

b) $\frac{2}{x+3} + \frac{4}{x-3} = \frac{x^2+11x+12}{x^2-9}$

$\{-2\}$

$\frac{2}{x+3} + \frac{4}{x-3} = \frac{x^2+11x+12}{(x-3)(x+3)}$

5

LCD $= (x+3)(x-3)$:

$2(x-3) + 4(x+3) = x^2+11x+12$

$2x - 6 + 4x + 12 = x^2+11x+12$

$0 = x^2 + 5x + 6$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x+2=0$ or $x=-3$

$x=-2$

Check $x=-2$:

$LS = \frac{2}{1} + \frac{4}{-5}$

$= \frac{10}{5} - \frac{4}{5}$

$= \frac{6}{5}$

$RS = \frac{-6}{-5} = \frac{6}{5}$ ✓

$x=-3$ is extraneous

7. [5 points] Find the quotient and remainder of $\frac{9x^3-7x+7}{3x+4}$. Is $3x+4$ a factor of $9x^3-7x+7$?

$$
\begin{array}{r}
3x^2-4x+3 \\
3x+4\overline{)\,9x^3+0x^2-7x+7} \\
-(9x^3+12x^2) \\
\hline
-12x^2-7x+7 \\
-(-12x^2-16x) \\
\hline
9x+7 \\
-(9x+12) \\
\hline
-5
\end{array}
$$

(2)

Quotient: $3x^2-4x+3$

Remainder: -5

Factor? (Yes or No) No (remainder $\neq 0$)

(3)

8. [6 points] Xander and Yolanda are doing a 30 km run. Xander runs 4 km/h faster than Yolanda. It takes Yolanda two hours longer to complete the run. Find Xander's speed.

Let x = Xander's speed (km/h) (2)

$x-4$ = Yolanda's "

	speed (km/h)	distance (km)	time $(h = \frac{km}{km/h})$
Xander	x	30	$\frac{30}{x}$
Yolanda	$x-4$	30	$\frac{30}{x-4}$

Xander's time + 2 = Yolanda's time

$$\frac{30}{x}+2=\frac{30}{x-4}$$ (2)

LCD $= x(x-4)$: (1)

$$30(x-4)+2x(x-4)=30x$$
$$30x-120+2x^2-8x=30x$$
$$2x^2-8x-120=0$$
$$2(x^2-4x-60)=0$$
$$2(x-10)(x+6)=0$$ (1)

$x-10=0$ $\quad$ $x=-6$ ← Discard.

$x=10$

Xander's speed is 10 km/h.