Math 172–Quiz # 5

December 2, 2011 Name:_____

Instructors: Pat Wream & Leah Howard Total: 40 Points

1. [2 points] State the domain of the following rational expression in either set-builder notation or interval notation.

$\frac{x^2-1}{x^2+5x}$

$x^2+5x=0$

$x(x+5)=0$

$x=0,-5$

$\{x \mid x \neq 0 \text{ and } x \neq -5\}$

or: $(-\infty,-5)\cup(-5,0)\cup(0,\infty)$

2. [3 points] Reduce the expression to lowest terms.

$\frac{4m-24n}{36mn^2-m^3} = \frac{4(m-6n)}{m(36n^2-m^2)}$

$= \frac{4(m-6n)}{m(6n-m)(6n+m)}$

$= \frac{-4}{m(6n+m)}$

$\frac{-4}{m(6n+m)}$

3. [8 points] Perform the indicated operations. Express your answer in lowest terms.

a) $\frac{2x^2+7x-15}{8x-12} \div \frac{x^2-25}{2}$

$= \frac{(x+5)(2x-3)}{4(2x-3)} \times \frac{2}{(x-5)(x+5)}$

$= \frac{1}{2(x-5)}$

Mult to -30

Add to 7

$-3, 10$

$2x^2-3x+10x-15$

$= x(2x-3)+5(2x-3)$

$= (x+5)(2x-3)$

$\frac{1}{2(x-5)}$

$\frac{24}{(z-2)(z+2)^2}$

(b) $\frac{6}{z^2-4} - \frac{6}{z^2+4z+4}$

$= \frac{6}{(z-2)(z+2)} - \frac{6}{(z+2)^2}$ LCD $= (z-2)(z+2)^2$

$= \frac{6(z+2)}{(z-2)(z+2)^2} - \frac{6(z-2)}{(z-2)(z+2)^2}$

$= \frac{6z+12-6z+12}{(z-2)(z+2)^2} = \frac{24}{(z-2)(z+2)^2}$

4. [4 points] Simplify the following complex fraction:

$\frac{a+14}{a-6}$

$\frac{(a-2)(a+2)\left(\frac{4}{a-2} - \frac{3}{a+2}\right)}{(a-2)(a+2)\left(\frac{1}{2}a - \frac{2}{a+2}\right)}$ LCD $= (a-2)(a+2)$

$= \frac{4(a+2) - 3(a-2)}{-1(a+2) + 2(a-2)}$

$\frac{a-2}{2-a} = -1$

$= \frac{4a+8-3a+6}{-a-2+2a-4}$

$= \frac{a+14}{a-6}$

5. [3 points] Solve the following equation for b.

$b = \frac{ac}{a-c}$

$abc\left(\frac{1}{a} + \frac{1}{b}\right) = \left(\frac{1}{c}\right)abc$ LCD $= abc$

$bc + ac = ab$

$bc - ab = -ac$

$b(c-a) = -ac$

$b = \frac{-ac}{c-a}$ or $\frac{ac}{a-c}$

6. [9 points] Find the solution set for the following equations:

a) $\frac{x+2}{12} = \frac{5}{x-2}$

$\{8, -8\}$

$(x+2)(x-2) = 60$

$x^2 - 4 = 60$

$x^2 - 64 = 0$

$(x-8)(x+8) = 0$

$x-8=0$ $\quad$ $x+8=0$

$x=8$ $\quad$ $x=-8$

Check $x=8$: $\quad LS = \frac{10}{12} = \frac{5}{6}$

$RS = \frac{5}{6}$ ✓

Check $x=-8$: $\quad LS = \frac{-6}{12} = \frac{-1}{2}$

$RS = \frac{5}{-10} = \frac{-1}{2}$ ✓

b) $\frac{3}{x+2} + \frac{3}{x-2} = \frac{x^2+8}{x^2-4}$

$\{4\}$

$(x-2)(x+2)\left[\frac{3}{x+2} + \frac{3}{x-2}\right] = \frac{x^2+8}{(x-2)(x+2)}(x-2)(x+2)$ $\quad LCD = (x-2)(x+2)$

$3(x-2) + 3(x+2) = x^2+8$

$3x-6+3x+6 = x^2+8$

$6x = x^2+8$

$0 = x^2 - 6x + 8$

$0 = (x-4)(x-2)$

$x-4=0$ $\quad$ $x-2=0$

$x=4$ $\quad$ $x=2$

Check $x=4$

$LS = \frac{3}{6} + \frac{3}{2} = \frac{1}{2} + \frac{3}{2} = 2$

$RS = \frac{16+8}{16-4} = \frac{24}{12} = 2$ ✓

Check $x=2$

$LS = \frac{3}{4} + \frac{3}{0}$ undefined

$x=2$ is extraneous

7. [5 points] Find the quotient and remainder of $\frac{8x^3-4x+4}{2x+1}$. Is $2x+1$ a factor of $8x^3-4x+4$?

$$\begin{array}{r} 4x^2-2x-1 \\ 2x+1 \overline{)\,8x^3+0x^2-4x+4} \\ -8x^3+4x^2 \\ \hline -4x^2-4x+4 \\ -(-4x^2-2x) \\ \hline -2x+4 \\ -(-2x-1) \\ \hline 5 \end{array}$$

Quotient: $4x^2-2x-1$

Remainder: 5

Factor? (Yes or No) No

Remainder $\neq 0$

8. [6 points] Amy and Brady are doing a 6km hike. Amy hikes 2km/h faster than Brady does. If it takes Brady 30 minutes longer to finish the hike, find both their speeds.

Let a = Amy's speed (km/h)

$a-2$ = Brady's speed

Time $\left(\frac{km}{km/h}\right)$

$\frac{30}{60}$

$$\frac{6}{a} = \frac{6}{a-2} - \frac{1}{2}$$

Amy's time Brady's time

LCD $= 2a(a-2)$

$$2a(a-2)\frac{6}{a} = \left(\frac{6}{a-2} - \frac{1}{2}\right)2a(a-2)$$

$$12(a-2) = 12a - a(a-2)$$

$$12a-24 = 12a - a^2+2a$$

$$a^2-2a-24 = 0$$

$$(a-6)(a+4) = 0$$

$a=6$ $a=-4$ No.

Check $a=6$

LS $= \frac{6}{6} = 1$

RS $= \frac{6}{4} - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = 1$ ✓

Amy's speed is 6 km/h

Brady's speed is 4 km/h.