Math 172–Assignment # 6 Name:______________________

1. Simplify each expression and write your answers with positive exponents. Do not convert your answer to radical form. Assume all variables represent positive real numbers.

a) $(-32)^{-\frac{3}{5}}$

$\frac{-1}{8}$ ______________

$= \left(\frac{1}{-32}\right)^{3/5}$

$= \left(\frac{1}{-2}\right)^{3}$

$= \frac{1}{-8}$

$= \frac{-1}{8}$

b) $\frac{a^{\frac{1}{2}}b^{2}}{(a^{\frac{1}{2}}b^{-\frac{3}{2}})^{2}}$

$\frac{b^5}{a^{1/2}}$ ______________

$= \frac{a^{1/2}\, b^{2}}{a b^{-3}}$

$= a^{-1/2}\, b^{5}$

$= \frac{b^5}{a^{1/2}}$

2. Rationalize the denominators and leave any irrational expressions in simplified radical form. Assume all variables represent positive real numbers.

a) $\frac{7}{\sqrt{20}}$

$\frac{7\sqrt{5}}{10}$ ______________

$= \frac{7}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{7\sqrt{5}}{10}$

b) $\sqrt[3]{\frac{5}{3x^2y}}$

$\frac{\sqrt[3]{45xy^2}}{3xy}$ ______________

$= \frac{\sqrt[3]{5}}{\sqrt[3]{3x^2y}} \cdot \frac{\sqrt[3]{9xy^2}}{\sqrt[3]{9xy^2}}$

$= \frac{\sqrt[3]{45xy^2}}{\sqrt[3]{27x^3y^3}}$

$= \frac{\sqrt[3]{45xy^2}}{3xy}$

3. Rewrite $\sqrt{1200m^9n^{25}}$, leaving it in simplified radical form. Assume all variables represent positive real numbers.

$\sqrt{1200m^9n^{25}} = \sqrt{400m^8n^{24}}\sqrt{3mn}$

$= 20m^4n^{12}\sqrt{3mn}$

$20m^4n^{12}\cdot\sqrt{3mn}$

4. Perform the following operations and leave any irrational expressions in simplified radical form. Assume all variables represent positive real numbers.

a) $\sqrt{12}-\sqrt{27}+\sqrt{48}$

$= 2\sqrt{3} - 3\sqrt{3} + 4\sqrt{3}$

$= 3\sqrt{3}$

$3\sqrt{3}$

b) $\sqrt[4]{162r^9t^{10}} \div \sqrt[4]{2r^5t^3}$

$= \dfrac{\sqrt[4]{162r^9t^{10}}}{\sqrt[4]{2r^5t^3}}$

$= \sqrt[4]{\dfrac{162r^9t^{10}}{2r^5t^3}}$

$= \sqrt[4]{81r^4t^7}$

$= \sqrt[4]{81r^4t^4}\,\sqrt[4]{t^3}$

$= 3rt\cdot\sqrt[4]{t^3}$

$3rt\cdot\sqrt[4]{t^3}$

c) $\sqrt[3]{81d^7} - \frac{\sqrt[3]{3d^{10}}}{d}$

$2d^2\cdot\sqrt[3]{3d}$

$= \sqrt[3]{27d^6}\,\sqrt[3]{3d} - \dfrac{\sqrt[3]{d^9}\,\sqrt[3]{3d}}{d}$

$= 3d^2\cdot\sqrt[3]{3d} - \dfrac{d^3\cdot\sqrt[3]{3d}}{d}$

$= 3d^2\cdot\sqrt[3]{3d} - d^2\cdot\sqrt[3]{3d}$

$= 2d^2\cdot\sqrt[3]{3d}$

d) $(2\sqrt{5}+3)^2-(2\sqrt{5}-3)^2$

$24\sqrt{5}$

$= (4(5) + 12\sqrt{5}+9) - (4(5) - 12\sqrt{5}+9)$

$= 20 + 12\sqrt{5}+9 \quad -20 + 12\sqrt{5} - 9$

$= 24\sqrt{5}$

e) $\left(\frac{3\sqrt{y}}{1-2\sqrt{y}}\right) \cdot \frac{(1+2\sqrt{y})}{(1+2\sqrt{y})}$

$\dfrac{3\sqrt{y}+6y}{1-4y}$

$= \dfrac{3\sqrt{y}+6(y)}{1-4(y)}$

$= \dfrac{3\sqrt{y}+6y}{1-4y}$

f) $(x^2\sqrt[3]{x^4})^{12}$

x^{40}

$= x^{24}(x^{4/3})^{12}$

$= x^{24} \cdot x^{16}$

$= x^{40}$

5. Find all real solutions to each equation.

a) $(2-b)^2 = 8$

$\{2-2\sqrt{2},\ 2+2\sqrt{2}\}$

$2-b = \pm\sqrt{8}$

$2-b = \pm 2\sqrt{2}$

$2-b = 2\sqrt{2}$ $\quad$ $2-b = -2\sqrt{2}$

$2-2\sqrt{2} = b$ $\quad$ $2+2\sqrt{2} = b$

b) $\sqrt[5]{5-3x} = 2$

$\{-9\}$

$5-3x = 2^5$

$5-3x = 32$

$-27 = 3x$

$-9 = x$

c) $\sqrt{x+10} - \sqrt{3-x} = 1$

$\{-1\}$

$\sqrt{x+10} = 1+\sqrt{3-x}$

SBS: $x+10 = \left(1+\sqrt{3-x}\right)^2$

$x+10 = 1+2\sqrt{3-x}+3-x$

$2x+6 = 2\sqrt{3-x}$

$x+3 = \sqrt{3-x}$

SBS: $x^2+6x+9 = 3-x$

$x^2+7x+6 = 0$

$(x+6)(x+1) = 0$

$x+6=0$ $\quad$ $x=-1$

$x=-6$

Check $x=-6$ ✗

$x=-1$ ✓

$x=-6$ is extraneous

6. Simplify the following expressions and write your answer in the form $a+bi$.

a) $(-5+3i)-(-4-2i)$ $\underline{-1+5i}$

$= -5+3i+4+2i$

$= -1+5i$

b) $\frac{(-5+i)}{(4+2i)} \cdot \frac{4-2i}{4-2i}$ $\underline{\frac{-9}{10}+\frac{7}{10}i}$

$= \frac{-20+10i+4i+2}{16+4}$

$= \frac{-18+14i}{20}$

$= \frac{-18}{20}+\frac{14}{20}i$

$= \frac{-9}{10}+\frac{7}{10}i$

7. Find the imaginary solutions to the following equations.

a) $3a^2+75=0$ $\underline{\{\pm 5i\}}$

$3a^2=-75$

$a^2=-25$

$a=\pm\sqrt{-25}$

$a=\pm 5i$

b) $2a^2+75=0$ $\underline{\left\{\frac{\pm 5i\sqrt{6}}{2}\right\}}$

$2a^2=-75$

$a^2=\frac{-75}{2}$

$a=\pm\sqrt{\frac{-75}{2}}$

$a=\pm\sqrt{-25}\,\frac{\sqrt{3}}{\sqrt{2}}$

$a=\pm 5i\,\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$a=\frac{\pm 5i\sqrt{6}}{2}$

8. Write $\sqrt[3]{2m}\times\sqrt[4]{3m}$ as a single radical. Leave your answer in radical form. $\underline{\sqrt[12]{432m^7}}$

$=(2m)^{1/3}(3m)^{1/4}$

$=(2m)^{4/12}(3m)^{3/12}$

$=(16m^4)^{1/12}(27m^3)^{1/12}$

$=\sqrt[12]{16m^4}\cdot\sqrt[12]{27m^3}$

$=\sqrt[12]{432m^7}$

$$\begin{array}{r} 27 \\ \times\ 16 \\ \hline 162 \\ 27\ \ \\ \hline 432 \end{array}$$