Math 172–Assignment # 5 **Name:**____________________

1. State the domain of the following rational expression in both interval notation and set-builder notation.

$\frac{2x+1}{x^3-4x}$

$(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

$\{x \mid x \neq 0,-2,2\}$

$$x^3-4x = x(x^2-4)$$
$$= x(x-2)(x+2)$$

3

2. Perform the indicated operations and simplify, using positive exponents.

a) $\frac{d^2-9}{10d^3} \div \frac{6-2d}{5d}$

$\frac{-(d+3)}{4d^2}$

$$= \frac{\cancel{(d-3)}^{-1}(d+3)}{\cancel{10d^3}_{2d^2}} \times \frac{\cancel{5d}}{2\cancel{(3-d)}}$$

$$= \frac{-(d+3)}{4d^2}$$

3

b) $\frac{\frac{5}{x-2}-\frac{4}{x+2}}{\frac{5}{x^2-4}-1}$

$\frac{x+18}{(3-x)(3+x)}$

$$= \frac{\left[\frac{5}{x-2}-\frac{4}{x+2}\right]}{\left[\frac{5}{(x-2)(x+2)}-1\right]} \cdot \frac{(x-2)(x+2)}{(x-2)(x+2)}$$

$$= \frac{5(x+2)-4(x-2)}{5-(x-2)(x+2)}$$

$$= \frac{5x+10-4x+8}{5-(x^2-4)}$$

$$= \frac{x+18}{5-x^2+4}$$

$$= \frac{x+18}{9-x^2}$$

$$= \frac{x+18}{(3-x)(3+x)}$$

3

$\frac{a-4}{(a-1)(a-2)}$

c) $\frac{3}{a-1} - \frac{2}{a-2}$

$= \frac{3(a-2)}{(a-1)(a-2)} - \frac{2(a-1)}{(a-1)(a-2)}$

$= \frac{3a-6 \quad -2a+2}{(a-1)(a-2)}$

$= \frac{a-4}{(a-1)(a-2)}$

3

$\frac{2n+m^3}{m^2n}$

d) $2m^{-2}+mn^{-1}$

$= \frac{2}{m^2} + \frac{m}{n}$

$= \frac{2n}{m^2n} + \frac{m^3}{m^2n}$

$= \frac{2n+m^3}{m^2n}$

3

$\frac{6}{x-2}$

e) $\frac{5}{x-2}+\frac{1}{x}+\frac{2}{x^2-2x}$

$= \frac{5}{x-2} + \frac{1}{x} + \frac{2}{x(x-2)}$

$= \frac{5x}{x(x-2)} + \frac{x-2}{x(x-2)} + \frac{2}{x(x-2)}$

$= \frac{5x+x-2+2}{x(x-2)}$

$= \frac{6x}{x(x-2)}$

$= \frac{6}{x-2}$

4

$\frac{-k}{k-4}$

f) $\frac{k^2+2k-15}{k^2-4k+3} \div \frac{k^2+k-20}{k-k^2}$

$= \frac{\cancel{(k+5)}\cancel{(k-3)}}{\cancel{(k-3)}\cancel{(k-1)}} \times \frac{k\overset{-1}{\cancel{(1-k)}}}{\cancel{(k+5)}(k-4)}$

$= \frac{-k}{k-4}$

3

$\frac{5}{y^x}$

g) $\frac{10y^x-5}{y^{2x}-2y^x} \div \frac{2y^{2x}+7y^x-4}{y^{2x}+2y^x-8}$

4

$= \frac{5\cancel{(2y^x-1)}}{y^x\cancel{(y^x-2)}} \times \frac{\cancel{(y^x+4)}\cancel{(y^x-2)}}{\cancel{(2y^x-1)}\cancel{(y^x+4)}}$

$= \frac{5}{y^x}$

Rough work:

$2y^{2x}+7y^x-4$

$= 2y^{2x}+8y^x-y^x-4$

$= 2y^x(y^x+4)-1(y^x+4)$

$= (2y^x-1)(y^x+4)$

3. Find the quotient and the remainder.

$(x^3-x^2-10x-8) \div (x-4)$

$$
\begin{array}{r}
x^2+3x+2 \\
x-4 \;\big)\; x^3-x^2-10x-8 \\
-\;x^3-4x^2 \\
\hline
3x^2-10x-8 \\
-\;3x^2-12x \\
\hline
2x-8 \\
-\;2x-8 \\
\hline
0
\end{array}
$$

Quotient: x^2+3x+2

Remainder: 0

3

$2 + \frac{6}{x-3}$

4. Rewrite $\frac{2x}{x-3}$ in the form quotient + $\frac{\text{remainder}}{\text{divisor}}$

$$\begin{array}{r} 2 \\ x-3 \;\big)\; 2x + 0 \\ -\;\; 2x - 6 \\ \hline 6 \end{array}$$

②

$$\frac{2x}{x-3} = 2 + \frac{6}{x-3}$$

5. Use division to determine whether $x^2 - 2$ is a factor of $2x^4 + x^3 - 2x - 10$. Explain your reasoning.

④

$$\begin{array}{r} 2x^2 + x + 4 \\ x^2-2 \;\big)\; 2x^4 + x^3 + 0x^2 - 2x - 10 \\ -(2x^4 \quad\quad -4x^2) \\ \hline x^3 + 4x^2 - 2x - 10 \\ -(x^3 \quad\quad -2x) \\ \hline 4x^2 - 10 \\ -(4x^2 - 8) \\ \hline -2 \end{array}$$

$\boxed{\text{No, the remainder is not 0.}}$

6. Solve the following equations.

a) $\frac{7}{x} = \frac{x}{7}$

$\{-7, 7\}$

③

$49 = x^2$

$49 - x^2 = 0$

$(7-x)(7+x) = 0$

$7 - x = 0$ $\quad$ $x = -7$

$x = 7$

Check: $x = 7$ ✓

$x = -7$ ✓

b) $\frac{3}{y-2}+\frac{2y}{4-y^2}=\frac{5}{y+2}$

$\{4\}$ 3

$$\frac{3}{y-2}+\frac{2y}{(2-y)(2+y)}=\frac{5}{y+2} \quad \boxed{LCD=(y-2)(y+2)}$$

$$(y-2)(y+2)\left[\frac{3}{y-2}+\frac{2y}{(2-y)(2+y)}\right]=\left[\frac{5}{y+2}\right](y-2)(y+2)$$

$$3(y+2)+2y\frac{\cancel{(y-2)}^{-1}}{\cancel{(2-y)}}=5(y-2)$$

$$3(y+2)-2y=5(y-2)$$

$$3y+6-2y=5y-10$$

$$16=4y$$
$$4=y$$

Check $y=4$ ✓

c) $\frac{x}{x+5}+\frac{5}{x}=\frac{25}{x^2+5x}$

$\emptyset$

$$\frac{x}{x+5}+\frac{5}{x}=\frac{25}{x(x+5)}$$

4

$$x(x+5)\left[\frac{x}{x+5}+\frac{5}{x}\right]=\left[\frac{25}{x(x+5)}\right]x(x+5)$$

$$x(x)+5(x+5)=25$$

$$x^2+5x+25=25$$

$$x^2+5x=0$$

$$x(x+5)=0$$

$x=0 \qquad x+5=0$

$\qquad\quad x=-5$

Check: $x=0$ X

$x=-5$ X

No solution.

$P = \dfrac{A}{1+rt}$

(3)

7. Solve the following equation for P:
$t = \frac{A-P}{Pr}$

$Prt = \frac{A-P}{Pr}(Pr)$

$Prt = A-P$

$P + Prt = A$

$P(1+rt) = A$

$P = \dfrac{A}{1+rt}$

8. It takes a small pump 5 minutes longer than a large pump to pump up an inflatable boat. If both pumps are working together, they can pump up the boat in only 6 minutes. How long does it take each pump working by itself?

(1) Let x = large pump's time (minutes)
$x+5$ = small "

Large pump: $\frac{1}{x}$ boats/min

Small pump: $\frac{1}{x+5}$ boats/min

Together: $\frac{1}{6}$ boats/min

(1) $\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$

$6(x)(x+5)\left[\frac{1}{x} + \frac{1}{x+5}\right] = \frac{1}{6}(6)x(x+5)$

(1) $6(x+5) + 6x = x(x+5)$

$6x + 30 + 6x = x^2 + 5x$

$0 = x^2 - 7x - 30$

$(x-10)(x+3) = 0$

$x = 10 \qquad x = -3$

Time is positive, so $x = 10$.

Check $x=10$ ✓

Large pump takes 10 mins; small pump takes 15 mins. (1)