Math 172–Assignment # 4 **Name:**_____

1. Perform the following computation and write your answer in scientific notation.

$\frac{(1.5\times10^{-12})(3\times10^{15})}{5\times10^{45}}$

9×10^{-43}

$= \frac{4.5\times10^{3}}{5\times10^{45}} = 9\times10^{-43}$

$= \frac{45\times10^{2}}{5\times10^{45}}$

(2)

2. Simplify each expression. Assume all variables represent nonzero real numbers. Write your answer with positive exponents.

a) $\frac{21y^{-5}z^{-3}}{-3xy^{7}z^{-7}}$

$\frac{-7z^{4}}{xy^{12}}$

$= \frac{-7y^{-12}z^{4}}{x}$

$= \frac{-7z^{4}}{xy^{12}}$

(2)

b) $\left(\frac{-3x^{-2}y}{2xy^{2}}\right)^{-4}$

$\frac{16x^{12}y^{4}}{81}$

$= \left(\frac{2xy^{2}}{-3x^{-2}y}\right)^{4} = \frac{2^{4}x^{4}y^{8}}{(-3)^{4}x^{-8}y^{4}} = \frac{16x^{4}y^{8}}{81x^{-8}y^{4}} = \frac{16x^{12}y^{4}}{81}$

(3)

c) $\left(-\frac{8a^{-8}b^{2}}{3b^{-5}}\right)^{-1}$

$\frac{-3a^{8}}{8b^{7}}$

$= -\frac{3b^{-5}}{8a^{-8}b^{2}}$

$= \frac{-3b^{-7}}{8a^{-8}} = \frac{-3a^{8}}{8b^{7}}$

(2)

d) $\left(\frac{7^{q-7}}{7^{3-4q}}\right)^{2}$

7^{10q-20}

$= \frac{7^{2q-14}}{7^{6-8q}}$

$= 7^{10q-20}$

(2)

3. Simplify.

a) $\frac{3^{-1}-9^{-1}}{18^{-1}}$

$= 18\left(\frac{1}{3}-\frac{1}{9}\right)$

$= 6-2$

$= 4$

4 (2)

b) $(1+2^{-2})^{-1}$

$= \left(1+\frac{1}{4}\right)^{-1}$

$= \left(\frac{5}{4}\right)^{-1}$

$= \frac{4}{5}$

$\frac{4}{5}$ (2)

4. Factor each polynomial completely.

a) $9x^3y-6x^2y+xy$

$= xy(9x^2-6x+1)$

$= xy(3x-1)^2$

$xy(3x-1)^2$ (2)

b) $3p^2+8pq-3q^2$

$= 3p^2+9pq-pq-3q^2$

$= 3p(p+3q)-q(p+3q)$

$= (3p-q)(p+3q)$

$(3p-q)(p+3q)$ (2)

c) $81r^3-3t^3$

$= 3(27r^3-t^3)$

$= 3(3r-t)(9r^2+3rt+t^2)$

$3(3r-t)(9r^2+3rt+t^2)$ (3)

d) t^2+7t-5

t^2+7t-5 is prime
(doesn't factor)

t^2+7t-5 (1)

e) $x^2y^2+4y^2-x^2-4$

$= y^2(x^2+4) - 1(x^2+4)$

$= (y^2-1)(x^2+4)$

$= (y-1)(y+1)(x^2+4)$

$(y-1)(y+1)(x^2+4)$ (3)

f) $16m^{2m}-24m^m+9$

Let $a=m^m$: $= 16a^2-24a+9$

$= (4a-3)^2$

$= (4m^m-3)^2$

$(4m^m-3)^2$ (2)

g) $a^5-a^3+a^2-1$

$= a^3(a^2-1)+1(a^2-1)$

$= (a^3+1)(a^2-1)$

$= (a+1)(a^2-a+1)(a-1)(a+1)$

$= (a+1)^2(a-1)(a^2-a+1)$

$(a+1)^2(a-1)(a^2-a+1)$ (3)

5. Multiply out the following expressions.

a) $(x+5)(x^2-2x+10)$

$= x^3-2x^2+10x+5x^2-10x+50$

$= x^3+3x^2+50$

x^3+3x^2+50 (2)

b) $(3x-4y)(x+6)$

$= 3x^2+18x-4xy-24y$

$3x^2+18x-4xy-24y$ (2)

c) $(a^x+b^{2y})^2$

$= (a^x)^2+2a^xb^{2y}+(b^{2y})^2$

$= a^{2x}+2a^xb^{2y}+b^{4y}$

$a^{2x}+2a^xb^{2y}+b^{4y}$ (3)

6. Solve the following equations.

a) $y^3+3y^2-40y=0$

$\{0,-8,5\}$ (3)

$y(y^2+3y-40)=0$

$y(y+8)(y-5)=0$

$y=0$ $\quad y+8=0$ $\quad y=5$

$y=-8$

b) $(d^2-1)^2-11(d^2-1)+24=0$

$\{2,-2,3,-3\}$ (4)

Let $a=d^2-1$:

$a^2-11a+24=0$

$(a-3)(a-8)=0$

$(d^2-4)(d^2-9)=0$

$(d-2)(d+2)(d-3)(d+3)=0$

$d-2=0$ $\quad d=-2$ $\quad d=3$ $\quad d=-3$

$d=2$

c) $|x^2-2x-16|=8$

$\{6,-4,4,-2\}$ (4)

$x^2-2x-16=8$ $\qquad x^2-2x-16=-8$

$x^2-2x-24=0$ $\qquad x^2-2x-8=0$

$(x-6)(x+4)=0$ $\qquad (x-4)(x+2)=0$

$x-6=0$ $\quad x=-4$ $\qquad x=4$ $\quad x=-2$

$x=6$

7. Find all sets of three consecutive integers such that the sum of their squares is 50.

Let x = 1st integer

$x+1$ = 2nd "

$x+2$ = 3rd "

(1)

$x^2+(x+1)^2+(x+2)^2=50$ (1)

$x^2+x^2+2x+1+x^2+4x+4=50$

$3x^2+6x+5=50$

$3x^2+6x-45=0$

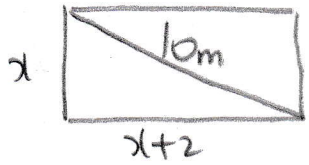
$3(x^2+2x-15)=0$

$3(x+5)(x-3)=0$ (1)

$x=-5$ $\quad$ $x=3$

There are two sets: $\{3,4,5\}$ and $\{-5,-4,-3\}$. (1)

8. You are designing a new swimming pool for Saanich Commonwealth Place. This rectangular pool is required to have a diagonal that is 10 metres long. You want the width of the pool to be 2 metres less than the length. Find the perimeter of the pool.



Let x = width of pool (m) (1)

$x+2$ = length of pool (m)

$x^2+(x+2)^2=10^2$ (1)

$x^2+x^2+4x+4=100$

$2x^2+4x-96=0$

$2(x^2+2x-48)=0$

$2(x+8)(x-6)=0$ (1)

$x=-8$ $\quad$ $x=6$

Width is positive, so $x=6$. Width = 6m and Length = 8m.

The perimeter is $2(6)+2(8)=28$ m (1)