

## 9.2, 9.3, 9.4 The Normal Distribution

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We'll look at data that is unimodal and symmetric.



Also called mound-shaped or bell-shaped.

Formally the shape is called the normal distribution.

FACT

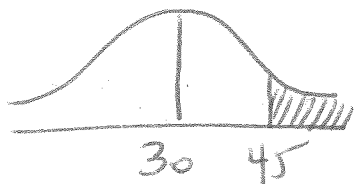
Probability = area under the curve.

For lectures and homework, use the online calculator:  
[davidmlane.com/hyperstat/z\\_table.html](http://davidmlane.com/hyperstat/z_table.html)

Test/Exam will have conceptual questions only.

Ex: The driving time between campuses is normally distributed with a mean of 30 mins and a standard deviation of 10 mins. Find the probability that the drive takes:

a) more than 45 mins



Use online calculator.

Select "Area from a value"  
Input mean and st. dev.  
Select range  
Hit "Recalculate"

0.0668

b) less than 25 mins

0.3085

c) between 40 and 50 mins

0.1359

d) exactly 40 mins

(between 40 and 40 mins)

0

Ex: a) A data set is normally distributed with a mean of 0 and a standard deviation of 1. Confirm the Empirical Rule by finding the probability that a measurement is within  $\mu - \sigma \leq x \leq \mu + \sigma$ .

Interval is  $-1 \leq x \leq 1$

0.68 ✓

b) Same but  $\mu - 2\sigma \leq x \leq \mu + 2\sigma$

Interval is  $-2 \leq x \leq 2$

0.95 ✓

c) Same but  $\mu - 3\sigma \leq x \leq \mu + 3\sigma$

Interval is  $-3 \leq x \leq 3$

0.997 ✓

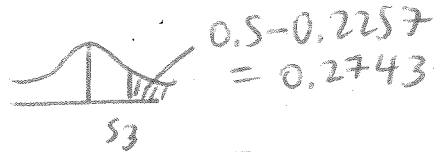
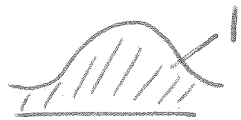
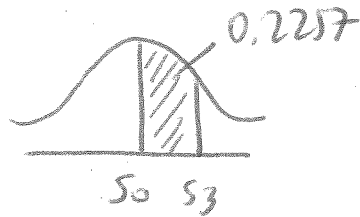
Ex: (Conceptual)

A data set is normally distributed with a mean of 50 and a standard deviation of 5. The probability that a measurement is between 50 and 53 is 0.2257.

Find the probability that a measurement is:

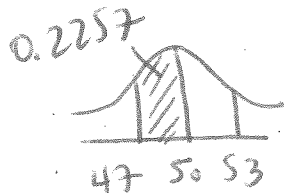
a) greater than 53

Given:



$0.2743$

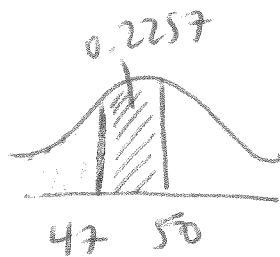
b) between 47 and 50



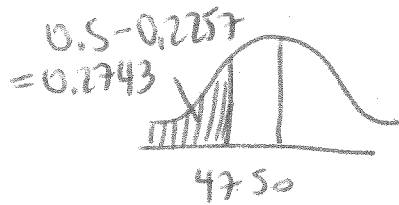
by symmetry

$0.2257$

c) less than 47

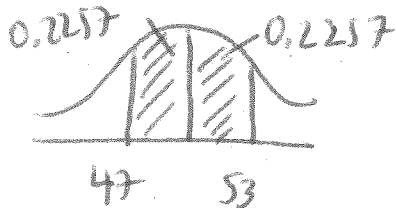


by symmetry



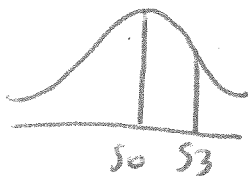
0.2743

d) between 47 and 53



0.4514

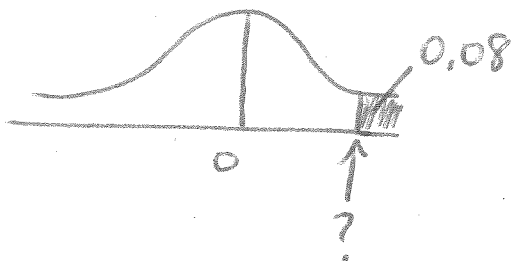
e) exactly 53



0

Ex: A data set is normally distributed with a mean of 0 and a standard deviation of 1.

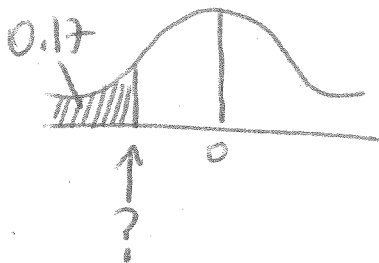
- a) Find the value that separates the highest 8% of measurements.



Select "Value from an area"  
Input area, mean and st. dev.  
Select above/below/between  
Hit "recalculate"

1.405

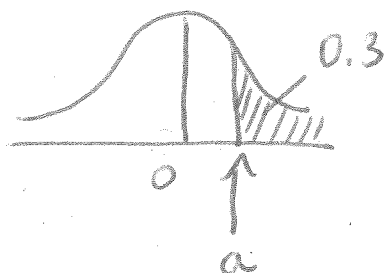
- b) Find the value that separates the lowest 17% of measurements.



-0.954

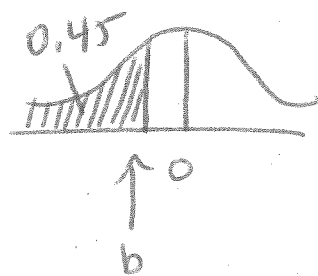
Ex: A data set is normally distributed with a mean of 0 and a standard deviation of 1.

a) Find  $a$  so that  $P(z > a) = 0.3$



$$a = 0.524$$

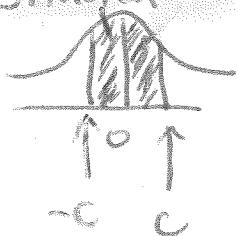
b) Find  $b$  so that  $P(z < b) = 0.45$



$$b = -0.126$$

c) Find  $c$  so that  $P(-c < z < c) = 0.3$

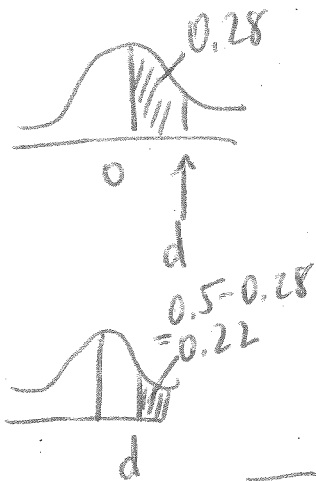
Total shaded area = 0.3



$$c = 0.385$$

Area = 0.3  
Select "between"

d) Find  $d$  so that  $P(0 < z < d) = 0.28$



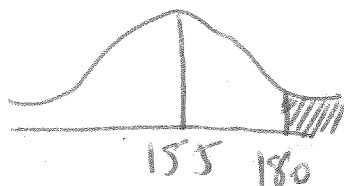
Area = 0.22  
Select "above"

$0.772$

Ex: The time it takes a student to write a Math 191 exam is normally distributed with a mean of 2 hours 35 mins and a standard deviation of 10 mins.

a) Find the probability that a random student will still be writing after 3 hours?

mean  $\mu = 155$  mins  
 $\sigma = 10$  mins



$0.0062$



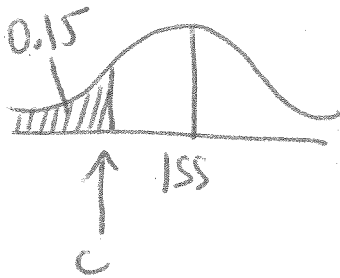
b) In a class of 500 students, how many students on average will still be writing after 3 hours?

$$P(\text{still writing}) = \frac{n(\text{still writing})}{500}$$

$$0.0062 = \frac{n(\text{still writing})}{500}$$

$$\begin{aligned}n(\text{still writing}) &= 500(0.0062) \\ &= 3.1 \\ &\approx 3\end{aligned}$$

c) The fastest 15% of students will complete the exam in  $c$  minutes. Find  $c$ .



$$c = 144.636 \text{ mins}$$