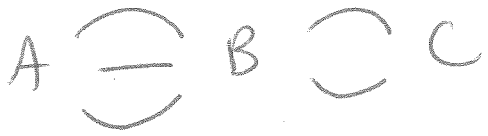
8.1 Counting Techniques

Multiplication Rule:

When performing a sequence of tasks, the number of possibilities is multiplied.

Ex:



# of possible routes from A to C?

$$\boxed{3} \times \boxed{2} = 6$$

- 3: # of routes from A to B
- 2: # of routes from B to C

Ex: How many 4-digit positive integers are divisible by 5?

$$\boxed{9} \times \boxed{10} \times \boxed{10} \times \boxed{2} = 1800$$

- 9: # of options for 1st digit (1-9)
- 10: 2nd digit (0-9)
- 10: 3rd digit (0-9)
- 2: 4th digit (0 or 5)

Ex: How many 4-digit PINs are possible if repetition of digits is not allowed?
(PIN = personal identification number)

10	×	9	×	8	×	7	= 5040
1st digit (0-9)		2nd digit		3rd digit		4th digit	

Ex: a) Suppose a license plate looks like ***###, where * is a capital letter and # is a digit 0-9. How many possible license plates are there?

$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$

b) What if the pattern can be ***### or ######*** ?

$17,576,000 + 17,576,000 = 35,152,000$

Complement Rule:

Number of Desirable Outcomes
= (Total Number of Outcomes) − (Number of Undesirable Outcomes)

Ex: Suppose a license plate looks like ***###, where * is a capital letter and # is a digit 0-9. The letter combination ABC is not allowed. How many possibilities are there?

$$\text{Total \#} = 26 \times 26 \times 26 \times 10 \times 10 \times 10$$
$$= 17,576,000$$

$$\text{\# That Start ABC} = 1 \times 1 \times 1 \times 10 \times 10 \times 10$$
$$= 1,000$$

$$\text{\# That Don't start ABC} = 17,576,000 - 1,000$$
$$= 17,575,000$$

Ex: How many 5-character case-sensitive alphanumeric passwords are there:

a) in total?

case-sensitive alphanumeric: 0, 1, ..., 9, a, b, ..., z, A, B, ..., Z
(62 symbols)

$$62 \times 62 \times 62 \times 62 \times 62 = 916,132,832$$
$$\text{or } 62^5$$

b) that contain at least one letter and at least one number?

with no letters = $10 \times 10 \times 10 \times 10 \times 10$
or 10^5

with no numbers = $52 \times 52 \times 52 \times 52 \times 52$
or 52^5

with at least one letter and at least one number = $62^5 - 10^5 - 52^5$
$= 535,828,800$

Let $n(A)$ = # ways A can happen

Inclusion-Exclusion Rule

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

Ex: Count how many integers from 1 to 15 are divisible by 3 or 5:

a) directly
3, 5, 6, 9, 10, 12, 15
7

b) using Inclusion-Exclusion
$n(\text{divisible by } 3) = 5$ (3, 6, 9, 12, 15)
$n(\text{divisible by } 5) = 3$ (5, 10, 15)
$n(\text{divisible by 3 and 5}) = 1$ (15)
$n(\text{divisible by 3 or 5}) = 5 + 3 - 1 = 7$

Ex: How many 4-digit PINs:

a) start with 9?

$1 \times 10 \times 10 \times 10 = 1000$

b) end with 4?

$10 \times 10 \times 10 \times 1 = 1000$

c) start with 9 and end with 4?

$1 \times 10 \times 10 \times 1 = 100$

d) start with 9 or end with 4?

$n(\text{start 9 or end 4}) = n(\text{start 9}) + n(\text{end 4}) - n(\text{start 9 and end 4})$

$= 1000 + 1000 - 100$

$= 1900$

e) start with 4 or 9?

$2 \times 10 \times 10 \times 10 = 2000$