

6.4 Measures of Relative Standing

The z-score is $z = \frac{x - \mu}{\sigma}$, where x is a measurement.

If working with a sample: $z = \frac{x - \bar{x}}{s}$.

Ex: You run a race and your time is 70 mins. The average time was 60 mins, with a standard deviation of 5 mins. Find the z-score of your time.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{70 - 60}{5} \\ &= 2 \end{aligned}$$

Your time was 2 standard deviations above the mean.

FACT

A z-score is the number of standard deviations above or below the mean.

Ex: Find the z-score of:

a) $x = \mu$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{\mu - \mu}{\sigma}$$

$$= 0$$

b) $x = \mu + 1.5\sigma$

$$z = \frac{(\mu + 1.5\sigma) - \mu}{\sigma}$$

$$= \frac{1.5\sigma}{\sigma}$$

$$= 1.5$$

c) $x = \mu - 3\sigma$

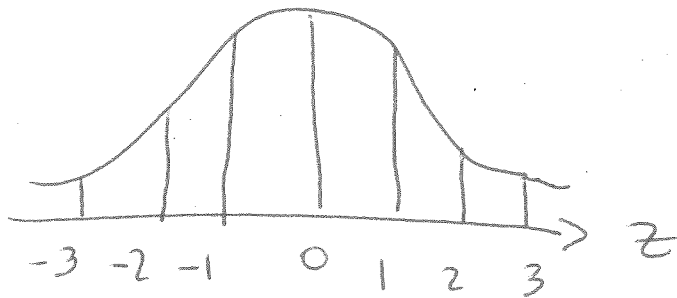
$$z = \frac{(\mu - 3\sigma) - \mu}{\sigma}$$

$$= \frac{-3\sigma}{\sigma}$$

$$= -3$$

Recall: Empirical Rule

If data is mound-shaped:



68% of data

95% of data

99.7% of data

FACT

z -scores bigger than 3 or less than -3 are considered outliers.

Ex: A student wrote two tests.

| | Course A | Course B |
|--------------------|----------|----------|
| Student's Mark | 74.5 | 90 |
| Average Mark | 70 | 78 |
| Standard Deviation | 1.5 | 5 |

In which course did the student do best relative to the class?

$$\begin{aligned}\text{Course A: } z &= \frac{x - \mu}{\sigma} \\ &= \frac{74.5 - 70}{1.5} \\ &= 3\end{aligned}$$

$$\begin{aligned}\text{Course B: } z &= \frac{x - \mu}{\sigma} \\ &= \frac{90 - 78}{5} \\ &= 2.4\end{aligned}$$

Course A