# 4.3 Logarithmic Growth

$\log_2 16 = 4$ $\quad\quad 2^{?} = 16$

$\log_2 2 = 1$ $\quad\quad 2^{?} = 2$

$\log_2 1 = 0$

$\log_{10} 100 = 2$

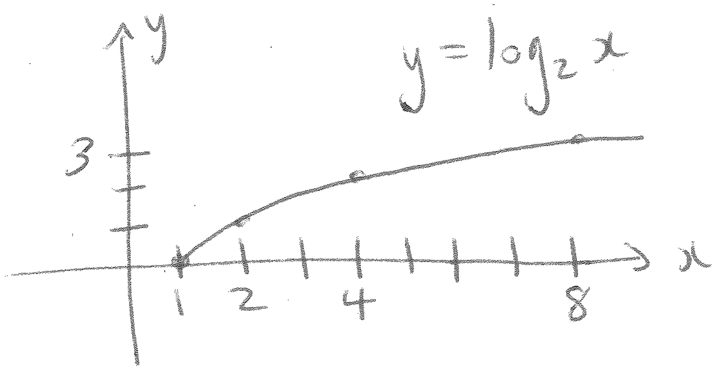
$\log_{10} 0.1 = -1$

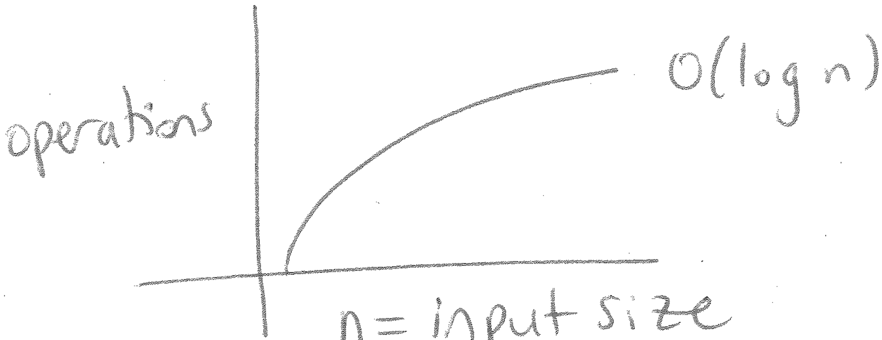
Notation: $\log n$ means $\log_{10} n$

$\log 100 = \log_{10} 100 = 2$

$\log 0.1 = \log_{10} 0.1 = -1$

| $x$ | $y = \log_2 x$ |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |
| 8 | 3 |






The graph is always increasing.
The function does not have a maximum value.

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Ordered List: 2, 5, 8, 9, 12, 17, 23, 40
In which position is 23?

Bad Algorithm
Check 1st position
Check 2nd position
⋮
Until desired number is found
$\boxed{\text{This algorithm is } O(n)}$

Quick Ex:
Is 23 in Position 1? No
" Position 2? No
⋮
" Position 6? No
" Position 7? YES

# Good Algorithm

1. Break list in two halves
2. Select lower half or upper half
3. Repeat Steps 1 and 2 until desired number is found.

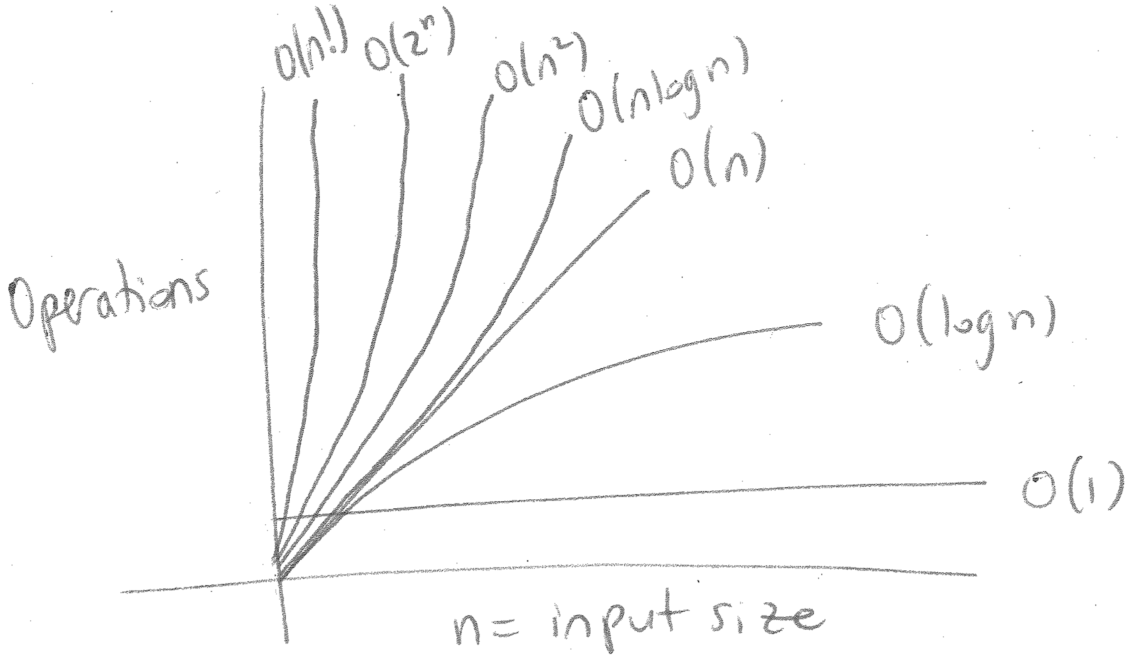
$\boxed{\text{This algorithm is } O(\log n).}$

Quick Ex:

~~2 5 8 9~~ 12 17 23 40 Positions 5-8

~~12 17~~ 23 40 Positions 7-8

23 ~~40~~ Position 7




Note: Curves may cross when n is small.

Ex: Order these from smallest to largest:

$O(n^2)$, $O(n\log n)$, $O(2^n)$, $O(n!)$, $O(1)$, $O(n)$, $O(\log n)$

$O(1)$, $O(\log n)$, $O(n)$, $O(n\log n)$, $O(n^2)$, $O(2^n)$, $O(n!)$

Ex: Find the order:

a) $3n + 4\log n$

$O(n)$

b) $(\log n)(3+2n)$

$= 3\log n + 2n\log n$

$O(n\log n)$

c) $3 + 4\log n$

$O(\log n)$