4.2 Factorial and Exponential Growth

Recall: Factorials

$3! = 3 \cdot 2 \cdot 1 = 6$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

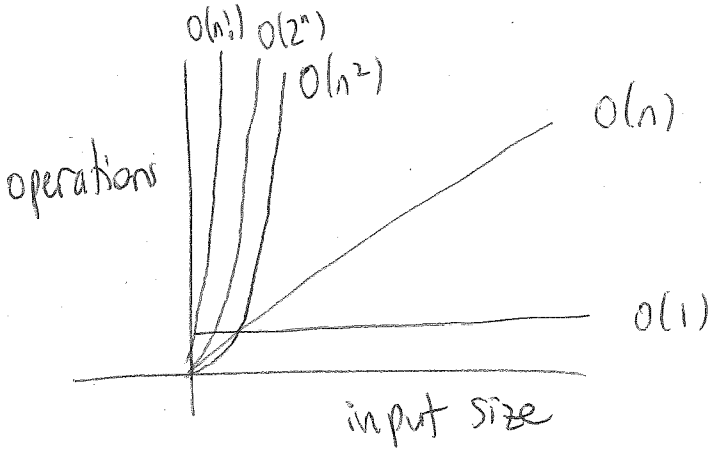
$n! = n(n-1)(n-2) \cdots 2 \cdot 1$

n	polynomial n^2	exponential 2^n	factorial $n!$
1	1	2	1
2	4	4	2
3	9	8	6
4	16	16	24
5	25	32	120
⋮			
50	2500	1.1×10^{15}	3.0×10^{64}

FACT

2^n is dominant over $1, n, n^2, n^3, n^4$ etc.

$n!$ is dominant over 2^n



Ex: Find the order of the expression

a) $9n^2 + 8(2^n)$

$O(2^n)$

b) $4!$

$O(1)$ because $4!$ is a constant.

c) $8 \cdot n!$

$O(n!)$

d) $20n(n+2)$

$= 20n^2 + 40n$

$O(n^2)$

e) $9n^2 + 3 \cdot n! + 8(2^n)$

$O(n!)$

Ex: Rank the following from smallest to largest: $O(n)$, $O(2^n)$, $O(n!)$, $O(1)$, $O(n^2)$

$O(1)$, $O(n)$, $O(n^2)$, $O(2^n)$, $O(n!)$

Ex: Label the curves