3.1 Sequences and Series

Sequence: ordered list of numbers

A sequence is finite if it has a final term.
A sequence is infinite otherwise.

Quick Ex: $2, 5, 8, \ldots, 32$ is a finite sequence
$2, 5, 8, \ldots$ is an infinite sequence

Ex: Identify the pattern

a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{256}$
multiply by $\frac{1}{2}$

b) $1, 4, 9, 16, \ldots$
squares

c) $5, -5, -15, \ldots$
subtract 10

A sequence can be indexed starting at any non-negative integer. Could be written:

$a_0, a_1, a_2, \ldots$
OR
$a_1, a_2, a_3, \ldots$
OR
$a_m, a_{m+1}, a_{m+2}, \ldots$

Three ways to define a sequence:

1) List a few terms.
2) Give a general formula for a_n in terms of n.
3) Give a recursive formula for a_n in terms of previous term(s).

$2, 5, 8, \ldots, 32$ — LIST

$a_n = 3n-1$ for $1 \le n \le 11$ — GENERAL FORMULA

$$\begin{cases} a_1 = 2 \\ a_n = 3 + a_{n-1} & \text{for } 2 \le n \le 11 \end{cases}$$ — RECURSIVE FORMULA

<u>Ex</u>: $a_n = 4n-1$ for $n \ge 1$

Find a_1, a_2 and a_{100}.

$a_1 = 3$

$a_2 = 7$

$a_{100} = 399$

Ex: $a_n = 2^n + 1$ for $0 \le n \le 3$

a) write all the terms

$a_0 = 2, \quad a_1 = 3, \quad a_2 = 5, \quad a_3 = 9$

b) How many terms are there?

(last index) − (first index) + 1

$= 3 - 0 + 1$

$= 4$

Ex: Give a general formula for:

a) $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \ldots$

$a_n = \sqrt{n}$ for $n \ge 1$

b) $1, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{10}$

$a_n = \sqrt{n}$ for $1 \le n \le 10$

Ex: Find the first four terms

a) $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 3 & \text{for } n \ge 2 \end{cases}$

$a_1 = 2, \quad a_2 = a_1 + 3 = 5, \quad a_3 = a_2 + 3 = 8,$

$a_4 = a_3 + 3 = 11$

b) $\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = a_{n-1} + a_{n-2}, & n \geq 3 \end{cases}$

$a_1 = 1, \quad a_2 = 1, \quad a_3 = a_2 + a_1 = 2,$

$a_4 = a_3 + a_2 = 3$

<u>Factorials</u>

$n! = n(n-1)(n-2)\cdots 1$

pronounced "n factorial"

$3! = 3(2)(1) = 6$

$2! = 2(1) = 2$

$1! = 1$

$0! = 1$ by definition

<u>Ex</u>: Write a recursive formula for

$1, 2, 6, 24, 120, 720, \ldots$

$\begin{cases} a_1 = 1 \\ a_n = n\, a_{n-1} & \text{for } n \geq 2 \end{cases}$

Series: sum of numbers

$5+15+25+\ldots+105$ is a finite series

$5+15+25+\ldots$ is an infinite series

S_k = sum of the first k terms of a series

S_∞ = sum of all terms of an infinite series

Ex: Consider $16+20+24+\ldots$
Find S_3 and S_5.

$S_3 = 16+20+24 = 60$

$S_5 = 16+20+24+28+32 = 120$

Note: The series $a_1+a_2+a_3+\ldots$
has $S_3 = a_1+a_2+a_3$

The series $a_0+a_1+a_2+\ldots$
has $S_3 = a_0+a_1+a_2$

Sigma Notation

$$\sum_{n=1}^{4} (3n+1) = \underset{(n=1)}{4} + \underset{(n=2)}{7} + \underset{(n=3)}{10} + \underset{(n=4)}{13}$$

$$= 34$$

Ex: Evaluate

a) $\sum_{i=0}^{2} 3^i = 1 + 3 + 9$

$= 13$

b) $\sum_{j=2}^{8} 7 = 7+7+7+7+7+7+7$

$= 49$

Ex: Write in sigma notation:

$\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \dots$

$\sum_{n=6}^{\infty} \frac{1}{n}$ OR $\sum_{n=1}^{\infty} \frac{1}{n+5}$

OR $\sum_{n=0}^{\infty} \frac{1}{n+6}$