2.7 The Conditional

Conditional statement:

$$p \to q$$

"p implies q"

"if p then q"

Ex: The following is true:
If Barney is a dog then he has four legs.
Answer Yes, No, or Maybe.

a) Barney is a dog. Does he have 4 legs?

Yes

b) Barney is not a dog. Does he have 4 legs?

Maybe

c) Barney has 4 legs. Is he a dog?

Maybe

d) Barney doesn't have 4 legs. Is he a dog?

No

Ex: The following is true:
If the client likes you then you get promoted.

Client Likes You	Promoted	Promise Kept?
No	No	Yes
No	Yes	Yes
Yes	No	No ←
Yes	Yes	Yes

Promise is broken

Truth table for $p \to q$:

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

Ex: Is $p \to q$ logically equivalent to $\sim p \vee q$?

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

Identical

Yes

Fact $\sim p \vee q \iff p \to q$

The converse of $p \to q$ is $q \to p$.

The inverse of $p \to q$ is $\sim p \to \sim q$.

The Contrapositive of $p \to q$ is $\sim q \to \sim p$.

Ex: Build a truth table for $p \to q$, $q \to p$, $\sim p \to \sim q$ and $\sim q \to \sim p$

p	q	$p \to q$	$q \to p$	$\sim p$	$\sim q$	$\sim p \to \sim q$	$\sim q \to \sim p$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	1	1	1	0	0	1	1

Identical

Identical

Fact

All logically equivalent:

$\sim p \vee q$

$p \to q$

$\sim q \to \sim p$

All logically equivalent:

$\sim q \vee p$

$q \to p$

$\sim p \to \sim q$

Ex: Consider the statement:

"If it's raining then it's cloudy."

Write the indicated statement. Is it logically equivalent to the original statement?

a) the converse

If it's cloudy then it's raining.

NO

b) the contrapositive

If it's not cloudy then it's not raining.

YES

c) the inverse

If it's not raining then it's not cloudy.

NO

Ex: Write the contrapositive of:

" If I live in Victoria or Vancouver
then I live in BC."

Statement: $(p \vee q) \rightarrow r$

Contrapositive: $\sim r \rightarrow \sim(p \vee q)$

$\sim r \rightarrow \sim p \wedge \sim q$

If I don't live in BC then I don't live in Victoria and I don't live in Vancouver.