2.6 More Laws of Logic

De Morgan's Laws

$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$ $\qquad\qquad$ $\overline{AB} = \overline{A} + \overline{B}$

$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$ $\qquad\qquad$ $\overline{A+B} = \overline{A}\,\overline{B}$

Quick Ex: $\overline{\overline{A} + \overline{B}} = \overline{\overline{A}}\,\overline{\overline{B}}$

Quick Ex: $\sim(p \wedge \sim q) \Leftrightarrow \sim p \vee \sim(\sim q)$

Distributive Laws

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ $\qquad\qquad$ $A(B+C) = AB + AC$

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $\qquad\qquad$ $A + BC = (A+B)(A+C)$

Ex: Rewrite using the distributive laws

a) $\overline{C}(A+C) = \overline{C}A + \overline{C}C$

b) $(A+B)(A+\overline{B}) = A + B\overline{B}$

c) $\overline{B} + \overline{A}\,\overline{C} = (\overline{B} + \overline{A})(\overline{B} + \overline{C})$

d) $\overline{AB}B + \overline{AB}C = \overline{AB}(B+C)$

Absorption Laws

$p \wedge (p \vee q) \Leftrightarrow p$ $\qquad$ $A(A+B) = A$

$p \wedge (\sim p \vee q) \Leftrightarrow p \wedge q$ $\qquad$ $A(\overline{A}+B) = AB$

$p \vee (p \wedge q) \Leftrightarrow p$ $\qquad$ $A + AB = A$

$p \vee (\sim p \wedge q) \Leftrightarrow p \vee q$ $\qquad$ $A + \overline{A}B = A+B$

Ex: Rewrite using the absorption laws

a) $\overline{C}(\overline{C}+A) = \overline{C}$

b) $\overline{C}(C+A) = \overline{C}A$

c) $AB + ABC = AB$

d) $\overline{AB} + ABC = \overline{AB} + C$

Ex: Simplify $(\sim p \vee \sim q) \wedge (p \vee \sim q)$

$\Leftrightarrow (\sim q \vee \sim p) \wedge (\sim q \vee p)$ Commutative (twice)

$\Leftrightarrow \sim q \vee (\sim p \wedge p)$ Distributive

$\Leftrightarrow \sim q \vee 0$ Complement

$\Leftrightarrow \sim q$ Identity

Ex: Simplify $AB(\overline{A} + \overline{B})$

$= AB\,\overline{AB}$ De Morgan's

$= 0$ Complement

Ex: Prove $\overline{A} + \overline{\overline{A}\,\overline{B}} = \overline{B \cdot 0}$

$\overline{A} + \overline{\overline{A}\,\overline{B}} = \overline{A} + \overline{\overline{A}} + \overline{\overline{B}}$ De Morgan's

$= \overline{A} + A + B$ Complement (twice)

$= 1 + B$ Complement

$= 1$ Identity

$= \overline{0}$ Definition of 0 and 1

$= \overline{B \cdot 0}$ Identity

Ex: Simplify $\bar{B}(\bar{A}+B)+\bar{A}(\bar{A}+B)$

$= \bar{B}(B+\bar{A}) + \bar{A}(\bar{A}+B)$ Commutative

$= \bar{B}\bar{A} + \bar{A}$ Absorption (twice)

$= \bar{A} + \bar{A}\bar{B}$ Commutative (twice)

$= \bar{A}$ Absorption