2.5 Laws of Logic

<u>Ex</u>: Show that $p \wedge 1 \Leftrightarrow p$

p	1	$p \wedge 1$
0	1	0
1	1	1

Identical

$$p \wedge 1 \Leftrightarrow p$$

In Boolean symbols we can write:

$$A \cdot 1 \Leftrightarrow A$$

or $A \cdot 1 = A$

There are four "Identity Laws"

Classical Symbols	Boolean Symbols
$p \wedge 1 \Leftrightarrow p$	$A \cdot 1 = A$
$p \vee 1 \Leftrightarrow 1$	$A + 1 = 1$
$p \wedge 0 \Leftrightarrow 0$	$A \cdot 0 = 0$
$p \vee 0 \Leftrightarrow p$	$A + 0 = A$

Ex: Simplify $(q \wedge 1) \vee (p \wedge 0)$

$$\Leftrightarrow q \vee (p \wedge 0) \quad \text{Identity}$$
$$\Leftrightarrow q \vee 0 \quad \text{Identity}$$
$$\Leftrightarrow q \quad \text{Identity}$$

Commutative Laws

$p \wedge q \Leftrightarrow q \wedge p$ $\qquad$ $AB = BA$

$p \vee q \Leftrightarrow q \vee p$ $\qquad$ $A+B = B+A$

Ex: Simplify $0 \wedge p$

$$\Leftrightarrow p \wedge 0 \quad \text{Commutative}$$
$$\Leftrightarrow p \quad \text{Identity}$$

(Can do Commutative law mentally)

Associative Laws

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$ $\qquad$ $(AB)C = A(BC)$

$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $\qquad$ $(A+B)+C = A+(B+C)$

Ex: Simplify $(p \wedge 1) \wedge 1$

$$\Leftrightarrow p \wedge (1 \wedge 1) \qquad \text{Associative}$$

$$\Leftrightarrow p \wedge 1 \qquad \text{Identity}$$

$$\Leftrightarrow p \qquad \text{Identity}$$

Idempotent Laws

$p \wedge p \Leftrightarrow p$ $\qquad$ $AA = A$

$p \vee p \Leftrightarrow p$ $\qquad$ $A + A = A$

Quick Ex: $(\sim p \wedge q) \vee (\sim p \wedge q) \Leftrightarrow (\sim p \wedge q)$

Quick Ex: $(A + \overline{B})(A + \overline{B}) = A + \overline{B}$

Complement Laws

$\sim(\sim p) \Leftrightarrow p$ $\qquad$ $\overline{\overline{A}} = A$

$p \wedge \sim p \Leftrightarrow 0$ $\qquad$ $A\overline{A} = 0$

$p \vee \sim p \Leftrightarrow 1$ $\qquad$ $A + \overline{A} = 1$

Quick Ex: $(p \vee q) \wedge \sim(p \vee q) \Leftrightarrow 0$

Quick Ex: $ABC + \overline{ABC} = 1$

<u>Ex</u>: Simplify

$((\sim p \vee 0) \wedge (q \vee \sim q)) \wedge (1 \vee r)$

$\Longleftrightarrow (\sim p \wedge (q \vee \sim q)) \wedge (1 \vee r)$ Identity

$\Longleftrightarrow (\sim p \wedge 1) \wedge (1 \vee r)$ Complement

$\Longleftrightarrow \sim p \wedge (1 \vee r)$ Identity

$\Longleftrightarrow \sim p \wedge 1$ Identity

$\Longleftrightarrow \sim p$ Identity

<u>Ex</u>: Simplify $(p \wedge \sim p) \vee (p \vee \sim p)$

$\Longleftrightarrow 0 \vee (p \vee \sim p)$ Complement

$\Longleftrightarrow 0 \vee 1$ Complement

$\Longleftrightarrow 1$ Identity

<u>Ex</u>: Simplify $\sim (p \vee (q \wedge \sim r)) \wedge (p \vee (q \wedge \sim r))$

$\Longleftrightarrow 0$ Complement

Ex: Simplify $A(\bar{B}B) + B(A+\bar{A})$

$= A\cdot 0 + B(A+\bar{A})$ Complement

$= A\cdot 0 + B\cdot 1$ Complement

$= 0 + B$ Identity (twice)

$= B$ Identity

Ex: Show that $A\cdot 1 + B\bar{B} = \overline{\bar{A}\cdot 1}$

$A\cdot 1 + B\bar{B} = A\cdot 1 + 0$ Complement

$= A + 0$ Identity

$= A$ Identity

$= \bar{\bar{A}}$ Complement

$= \overline{\bar{A}\cdot 1}$ Identity

Ex: Show that $(p \wedge \sim p) \wedge \sim q \Leftrightarrow p \wedge (q \wedge \sim q)$

$$
\begin{aligned}
(p \wedge \sim p) \wedge \sim q &\Leftrightarrow 0 \wedge \sim q && \text{Complement} \\
&\Leftrightarrow 0 && \text{Identity} \\
&\Leftrightarrow p \wedge 0 && \text{Identity} \\
&\Leftrightarrow p \wedge (q \wedge \sim q) && \text{Complement}
\end{aligned}
$$