2.4 Boolean Expressions and Gate Representations

Various coding languages use various symbols for NOT, AND, OR, and logical proposition.

Classical symbols	Boolean symbols
p, q, r	A, B, C
$\sim p$	$\overline{A}$
$p \wedge q$	AB (or $A \cdot B$)
$p \vee q$	$A + B$

Ex: Write as a Boolean expression:

$$p \wedge (q \vee \sim r)$$

$$A(B + \overline{C})$$

Order of Operations

NOT, then AND, then OR

Brackets override the order.

The negation bar also acts like brackets: do the operation under the bar then the negation.

Ex: What is the order of operations?

a) $\bar{A}B+C$ — NOT, AND, OR

b) $A+\overline{BC}$ — AND, NOT, OR

c) $\overline{A+B}C$ — OR, NOT, AND

d) $(A+\bar{B})C$ — NOT, OR, AND

Ex: Is $\overline{A+B}$ logically equivalent to $\bar{A}+\bar{B}$?

A	B	A+B	$\overline{A+B}$	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	1
1	0	1	0	0	1	1
1	1	1	0	0	0	0

Not identical

No. $\overline{A+B} \not\Leftrightarrow \bar{A}+\bar{B}$

Ex: Simplify $AB + A\bar{B}$

A	B	AB	$\bar{B}$	$A\bar{B}$	$AB + A\bar{B}$
0	0	0	1	0	0
0	1	0	0	0	0
1	0	0	1	1	1
1	1	1	0	0	1

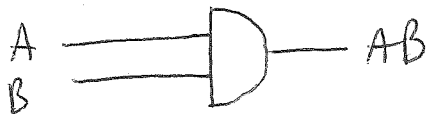
$AB + A\bar{B} \Leftrightarrow A$

Ex: Build the truth table for $A + \overline{B+C}$

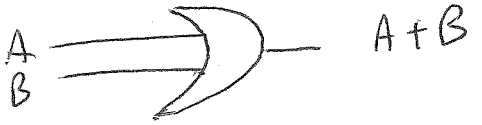
A	B	C	$B+C$	$\overline{B+C}$	$A + \overline{B+C}$
0	0	0	0	1	1
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	0	1

A <u>gate representation</u> is a way to visualize logical expressions.

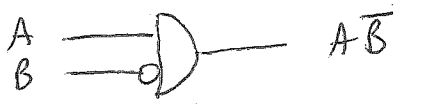
AND



OR



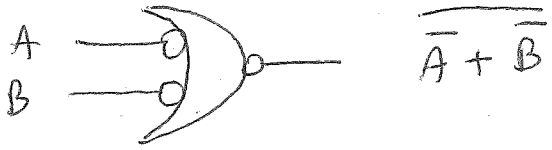
NOT An open circle is used for negation.



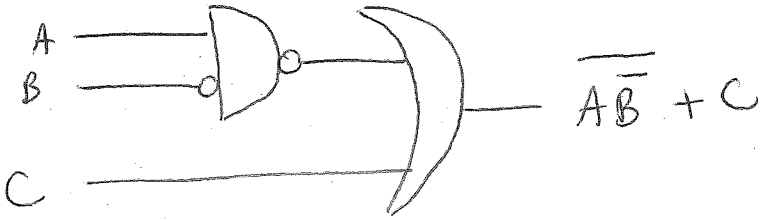
Ex: Draw the gate representation:

a) $A + \overline{B}$

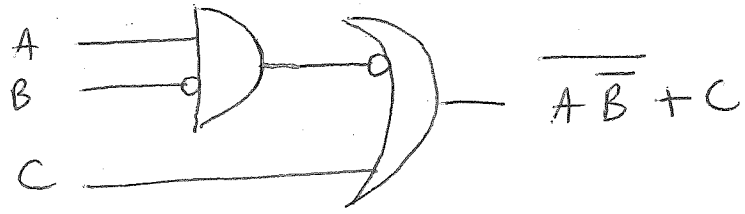
b) $\overline{\overline{A} + \overline{B}}$



c) $\overline{A\overline{B}} + C$



Also acceptable:



d) $A + \overline{B}\,\overline{C}$