# 10.2 Large Sample Confidence Intervals for the Mean

Point estimate for $\mu$: $\mu \approx \bar{x}$

Confidence intervals are better because we can select a specific confidence level.

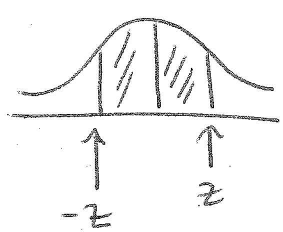
95% Confidence Interval for $\mu$:

" The average temperature in downtown Victoria at noon yesterday was between 13 and 15°C."

Notation:

$z$ is a variable that is normally distributed with $\mu = 0$ and $\sigma = 1$.

Ex: Consider the shaded area below:




Find the value of $z$ so that the shaded area is:

a) 0.90

Value from an area
Enter area = 0.90, $\mu = 0, \sigma = 1$
Select "between"
Hit "recalculate"

$z = 1.645$

b) 0.95

$z = 1.96$

c) 0.98

$z = 2.326$

d) 0.99

$z = 2.576$

The $z$ value is sometimes written $z_{\alpha/2}$ but we'll just write $z$.

## Confidence Interval Formula

$$\mu = \bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

Comments: $n$ is the sample size (# of measurements in the sample)

$n \geq 30$ is required

can use $s$ instead of $\sigma$

| Confidence Level | $z$ |
|---|---|
| 0.90 | 1.645 |
| 0.95 | 1.96 |
| 0.98 | 2.326 |
| 0.99 | 2.576 |

Ex: 40 students were asked how much they studied the weekend before exams. The mean was 15.1 hours with a standard deviation of 6.5 hours. Find:

a) a 90% confidence interval for $\mu$

$$\mu = \bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

$$= 15.1 \pm 1.645 \left(\frac{6.5}{\sqrt{40}}\right)$$

$$= 15.1 \pm 1.6906$$

$$= 15.1 \pm 1.7 \text{ hours}$$

or $$13.4 \le \mu \le 16.8 \text{ hours}$$

b) a 99% confidence interval for $\mu$

$$\mu = \bar{x} \pm z\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$= 15.1 \pm 2.576 \left(\frac{6.5}{\sqrt{40}}\right)$$

$$= 15.1 \pm 2.6 \text{ hours}$$

or $$12.5 \le \mu \le 17.7 \text{ hours}$$

## Meaning of 95% confidence:

95% of all possible samples lead to a confidence interval that contains the true value of $\mu$.

Ex: (Conceptual)

The accepted value of $\mu$ is 4.15. Researchers recently found a 95% confidence interval for $\mu$ to be $4 \le \mu \le 5$. Does this support the accepted value?

Yes. Accepted value is in the confidence interval.

Ex: (Conceptual)

Two research groups built 95% confidence intervals for $\mu$.

Research Group A: $2 \le \mu \le 2.89$

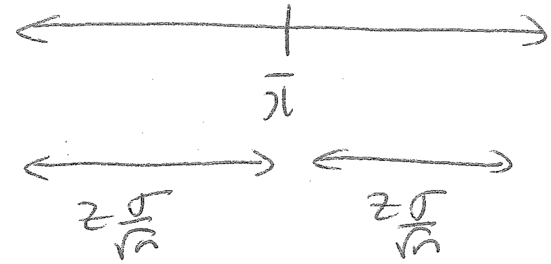
" B: $2.75 \le \mu \le 3.64$

Is it possible that both groups are correct?

Yes. The intervals overlap.

Consider $\mu = \bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

Visually:




$z\frac{\sigma}{\sqrt{n}}$ is called the <u>margin of error</u>.

<u>Ex</u>: We want to estimate the mean time between failures for a certain brand of hard drive. Historical data tells us $\sigma \approx 200$ hours. Find the minimum sample size so that a 99% confidence interval has error $\leq 50$ hours.

$$\frac{z\sigma}{\sqrt{n}} \leq 50$$

$$\frac{2.576\,(200)}{\sqrt{n}} \leq 50$$

$$2.576\,(200) \leq 50\sqrt{n}$$

$$\frac{2.576\,(200)}{50} \leq \sqrt{n}$$

Square both sides:

$$\left[\frac{2.576\,(200)}{50}\right]^2 \leq n$$

$$n \geq 106.17$$

Smallest $n$ is $n = 107$.

Ex: Name two ways to decrease the margin of error.

1) Increase sample size.

2) Decrease confidence level.