

$$(17) ((q \wedge p) \wedge \sim(p \wedge q)) \vee (p \wedge p)$$

$$\Leftrightarrow ((p \wedge q) \wedge \sim(p \wedge q)) \vee (p \wedge p) \quad \text{Commutative}$$

$$\Leftrightarrow 0 \vee (p \wedge p) \quad \text{Complement}$$

$$\Leftrightarrow p \wedge p \quad \text{Identity}$$

$$\Leftrightarrow p \quad \text{Idempotent}$$

$$(18) (BA + \overline{CB} CB) + \overline{\overline{AB}}$$

$$\Leftrightarrow (BA + 0) + \overline{\overline{AB}} \quad \text{Complement}$$

$$\Leftrightarrow (BA + 0) + AB \quad \text{Complement}$$

$$\Leftrightarrow BA + AB \quad \text{Identity}$$

$$\Leftrightarrow AB + AB \quad \text{Commutative}$$

$$\Leftrightarrow AB \quad \text{Idempotent}$$

$$(19) \quad \sim(p \vee q) \wedge (q \vee (q \wedge \sim r))$$

$$\Leftrightarrow \sim(p \vee q) \wedge q$$

Absorption

$$\Leftrightarrow (\sim p \wedge \sim q) \wedge q$$

De Morgan's

$$\Leftrightarrow \sim p \wedge (\sim q \wedge q)$$

Associative

$$\Leftrightarrow \sim p \wedge 0$$

Complement

$$\Leftrightarrow 0$$

Identity

$$(20) \quad \overline{A \overline{B} (\overline{A} + \overline{B}) + \overline{A} C}$$

$$\Leftrightarrow (\overline{A} + \overline{\overline{B}}) (\overline{A} + \overline{B}) + \overline{A} C$$

De Morgan's

$$\Leftrightarrow (\overline{A} + B) (\overline{A} + \overline{B}) + \overline{A} C$$

Complement

$$\Leftrightarrow \overline{A} + B \overline{B} + \overline{A} C$$

Distributive

$$\Leftrightarrow \overline{A} + 0 + \overline{A} C$$

Complement

$$\Leftrightarrow \overline{A} + \overline{A} C$$

Identity

$$\Leftrightarrow \overline{A}$$

Absorption

(21) a) MAYBE

b) YES

c) No

d) MAYBE

(22)

a) Converse of original.

Not logically equivalent to original.

b) Contrapositive of original.

Logically equivalent to original.

c) Inverse of original.

Not logically equivalent to original.

(23)

a) No

b) YES

c) YES

d) No

(24)

a) $a_n = \frac{n+1}{n}$ for $1 \leq n \leq 9$

Alternatively:

$$a_n = \frac{n}{n-1} \quad \text{for } 2 \leq n \leq 10$$

b) Each term is the previous term - 3

$$\begin{cases} a_1 = 4 \\ a_n = a_{n-1} - 3 \quad \text{for } n \geq 2 \end{cases}$$

Alternatively:

$$\begin{cases} a_0 = 4 \\ a_n = a_{n-1} - 3 \quad \text{for } n \geq 1 \end{cases}$$

$$(25) \quad \frac{2}{5} + \frac{2}{6} + \frac{2}{7} + \dots + \frac{2}{13}$$

$$= \sum_{n=5}^{13} \frac{2}{n}$$

Some alternative answers:

$$\sum_{i=5}^{13} \frac{2}{i} \quad \text{OR} \quad \sum_{i=0}^8 \frac{2}{i+5} \quad \text{OR} \quad \sum_{n=0}^8 \frac{2}{n+5}$$

(26) Arithmetic sequence
with $a_1 = 23$ and $d = 8$

$$a_n = a_m + (n-m)d \quad \text{for } n \geq m$$

Sub $m=1$: $a_n = a_1 + (n-1)d$ for $n \geq 1$

$$a_n = 23 + (n-1)(8) \quad \text{"}$$

$$a_n = 23 + 8n - 8 \quad \text{"}$$

$$\boxed{a_n = 15 + 8n \quad \text{for } n \geq 1}$$

$$(27) \quad a) \quad a_n = a_m + (n-m)d \quad \text{for } n \geq m$$

Sub $m=1$:

$$a_n = a_1 + (n-1)d \quad \text{for } n \geq 1$$

$$a_n = 19 + (n-1)6 \quad \text{"}$$

$$\text{Sub } n=35: \quad a_{35} = 19 + 34(6)$$

$$a_{35} = 223$$

b) From above:

$$a_n = 19 + (n-1)6 \quad \text{for } n \geq 1$$

$$\text{Sub } a_n = 1243: \quad 1243 = 19 + (n-1)6$$

$$1224 = (n-1)6$$

$$204 = n-1$$

$$205 = n$$

$$\boxed{a_{205} = 1243}$$

(28)

$$\sum_{j=12}^{59} (7j-2)$$

$$= 82 + 89 + 96 + \dots + 411$$

Arithmetic Series

a_m a_n

$$\begin{aligned} k &= \# \text{ of terms} \\ &= n - m + 1 \\ &= 59 - 12 + 1 \\ &= 48 \end{aligned}$$

$$\begin{aligned} S_k &= \frac{k}{2} (a_m + a_n) \\ &= \frac{48}{2} (82 + 411) \\ &= 11832 \end{aligned}$$

(29)

Geometric Sequence
with $a_1 = 4$ and $r = -2$

$$\begin{cases} a_1 = 4 \\ a_n = -2a_{n-1} \quad \& n \geq 2 \end{cases}$$

30) Geometric Sequence
with $a_1 = 3^{16}$ $r = \frac{1}{3}$

$$a_n = a_m r^{n-m} \quad \text{for } n \geq m$$

Sub $n=12$, $m=1$:

$$\begin{aligned} a_{12} &= a_1 r^{11} \\ &= 3^{16} \left(\frac{1}{3}\right)^{11} \\ &= 243 \end{aligned}$$

31) Geometric series with
 $a_1 = -6$ $r = -\frac{1}{2}$

$$a) \quad S_k = \frac{a_m (1-r^k)}{1-r}$$

Sub $k=8$, $m=1$:

$$\begin{aligned} S_8 &= \frac{a_1 (1-r^8)}{1-r} \\ &= \frac{-6 \left(1 - \left(-\frac{1}{2}\right)^8\right)}{\left(1 + \frac{1}{2}\right)} \\ &\approx -3.98 \end{aligned}$$

b) $-1 < r < 1$ ✓

$$\begin{aligned} S_{\infty} &= \frac{a_n}{1-r} \\ &= \frac{a_1}{1-r} \\ &= \frac{-6}{\left(1 + \frac{1}{2}\right)} \\ &= -4 \end{aligned}$$

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- a) Program 1
- b) Program 2
- c) 50,000

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- | | | |
|----------------|------------------|------------|
| a) $O(n)$ | b) $O(n^2)$ | c) $O(1)$ |
| d) $O(n!)$ | e) $O(2^n)$ | f) $O(n!)$ |
| g) $O(\log n)$ | h) $O(n \log n)$ | i) $O(n)$ |

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- $O(n!)$, $O(2^n)$, $O(n^2)$,
 $O(n \log n)$, $O(n)$, $O(\log n)$, $O(1)$

35 a) Each user is an experimental unit.

b) Country	QUALITATIVE
Number of Logins	DISCRETE
Time Spent Playing	CONTINUOUS

36 a) Unimodal symmetrical

b) There are no outliers.

c) The range 1-1.5 kg is the mode.

d) $\frac{30}{100} = 30\%$

37 The vertical axis does not begin at 0.