Term: 2022 Name: Solution Set

Instructor: Patricia Wrean

MATH 156-X01
Practice Test 3A

Total = $\frac{\quad}{30}$

- **All of the work on this test must be your own.**
- You may use a scientific calculator. You may not use a calculator with graphing capability or a smartphone app.

GOOD LUCK!

1. (4 points) Consider the following.

$$a_n = (-4)^n \qquad \text{for } 0 \le n \le 16$$

(a) Calculate the first three terms: 1 , −4 , 16

(b) How many terms does this have? 17

$a_0 = (-4)^0 = 1$

$a_1 = (-4)^1 = -4$

$a_2 = (-4)^2 = 16$

$k = n - m + 1$

$= 16 - 0 + 1$

$= 17$

2. (6 points) For each of the following,

- specify whether it is arithmetic, geometric, or neither,
- give a formula for a_n, being sure to specify what values to use for the index, and
- draw a box around your formula for a_n

(a) $5, -15, 45, \ldots$ geometric

general formula:

$a_n = a_m r^{n-m}$

$a_n = 5(-3)^{n-1}$ for $n \ge 1$

or $a_n = 5(-3)^n$ for $n \ge 0$

recursive:

$a_0 = 5$

$a_n = -3a_{n-1}$ for $n \ge 1$

(b) $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots, \frac{21}{20}$

neither

$a_n = \frac{n+1}{n}$ for $1 \le n \le 20$

or

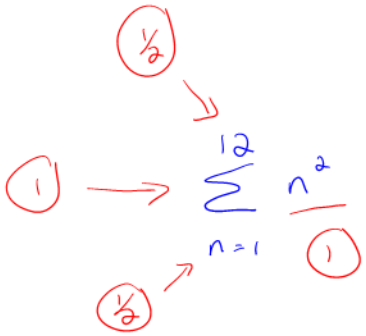
$a_n = \frac{n}{n-1}$ for $2 \le n \le 21$

didn't include endpoint or wrong number of terms $-\frac{1}{2}$

3. (3 points) Write the following sum in sigma notation. You do not need to calculate the total.

$$1 + 4 + 9 + 16 + \ldots + 144$$

$1^2 + 2^2 + 3^2 + 4^2 + \ldots + 12^2$



4. (5 points) Consider the following.

arithmetic with $d = -11$

$$27 + 16 + 5 + \ldots$$

(a) Is this a sequence or a series? Choose one: sequence / series (1)

(b) Calculate the ninety-ninth term. Show your work below. -1051

$a_n = a_m + (n-m)d$

$a_{99} = 27 + (99-1)(-11)$ (2)

$= -1051$

(c) Calculate the sum of the first ninety-nine terms. Show your work below. -50688

$S_k = \frac{k}{2}(a_m + a_n)$

$= \frac{99}{2}(27 + (-1051))$ (2)

$= -50688$

5. (3 points) Consider the arithmetic sequence with first term equal to 20 and fifteenth term equal to 104. What is the common difference for this sequence?

Let's use $m=1$

then $a_1 = 20$

$a_{15} = 104$

and $a_n = a_m + (n-m)d$ (1)

$104 = 20 + (15-1)d$ (1)

$84 = 14d$

$d = 6$

$d=6$ (1)

6. (2 points) For each of the following procedures, the number of operations needed for a task of size n is given below. Find Big O for each procedure.

(a) 4!

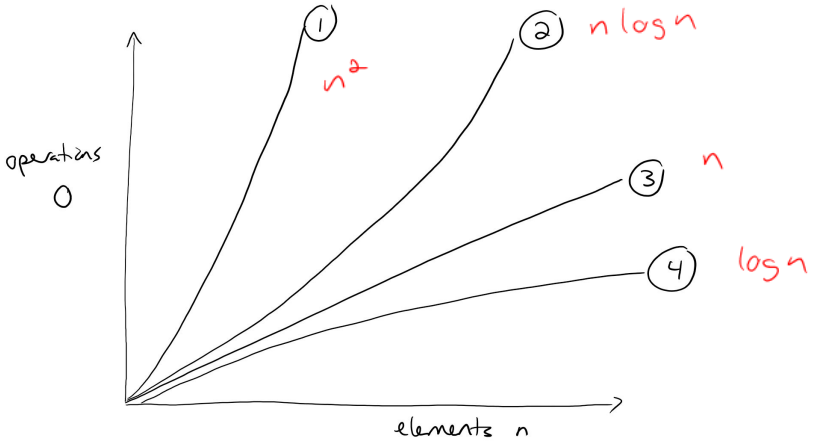
4! is a constant, so this is $O(1)$

(b) $n(n+2\log n+3)$ $= n^2 + 2n\log n + 3n$

grows the fastest

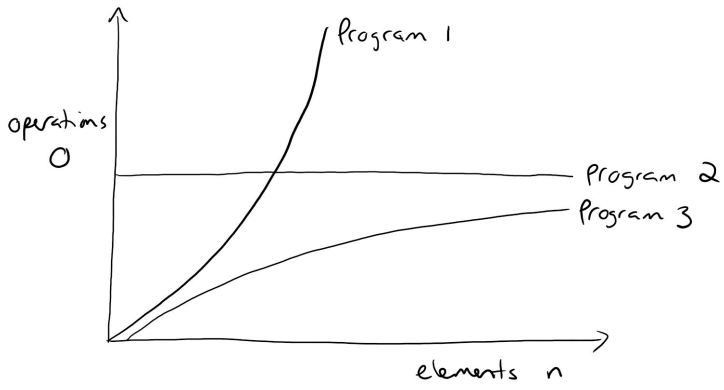
$O(n^2)$

7. (4 points) Match the Big O notation with its corresponding curve on the graph. Please note that the curves are labeled 1, 2, 3, and 4 going from left to right and that curve 3 is a straight line.



(a) $O(n \log n)$	2
(b) $O(\log n)$	4
(c) $O(n^2)$	1
(d) $O(n)$	3

8. (3 points) The following graph shows the number of operations O required to complete a task of size n for programs 1, 2, and 3. The number of operations required for Program 2 is a constant, so Program 2 is a horizontal straight line.



Indicate whether the following statements are true or false by selecting the correct answer.

(a) Program 3 is always more efficient than Program 1. (True) / False

(b) For very small n, Program 1 could be a better choice than Program 2. (True) / False

(c) For large n, the most efficient program is Program 3. True / (False)